

## MATHEMATICAL UNDERSTANDING OF CONSCIOUSNESS AND UNCONCIOUSNESS

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**ABSTRACT.** This paper approaches the subject of consciousness and unconsciousness from a mathematical point of view. It sets up a hypothesis that when unconscious state becomes conscious state, high density energy is released. We argue that the process of transformation of unconsciousness into consciousness can be expressed using the infinite recursive Heaviside step function. We claim that differentiation of the potential of unconsciousness with respect to time is the process of being conscious in a world where only time exists, since the thinking process never have any concrete space. We try to attribute our unconsciousness to a special solution of the multi-dimensional advection partial differential equation which can be represented by the finite recursive Heaviside step function. Mathematical language explains how the infinitive neural process is perceived and understood by consciousness in a definitive time.

### 1. INTRODUCTION

Human beings have tried to understand the world and themselves with numbers and images since the antiquity. Verbal language cannot embrace the whole scope of abstract world. Mathematics have been considered to represent different dimension of the world and mentality beyond language. The Pythagoreans believed that the principles of mathematics were the principles of all things [1]. Rene Descartes claimed that scientific knowledge should be built up with the fundamentals of mathematics and geometry [1]. Scientism tries to make believe that the evidence of real existence should be tested and validated only by numerical changes or visual and auditory proofs. Abstract or spiritual mental functions such as thinking, affect, and belief, which cannot be numerically positioned or physically tested, have been considered as something not-scientific and non-existent. Meanwhile, Nobel Laureate and atheist Peter

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McDawar said; That there is indeed a limit upon science is made very likely by the existence of questions that science cannot answer and that no conceivable advances of science would empower it to answer [2]. The classical physics and mathematics did not have any clue to relate brain mental function to numerical formula because mental function does not have any spatial dimension, but only having time dimension. Modern physicists solved this puzzle by confessing that the consciousness (and unconsciousness) of the person acts in the world but is not spatially extent [3]. While Freud and Jung empirically and intuitively inferred the existence of unconsciousness, contemporary modern physics made possible clarify the complex system of consciousness and unconsciousness after the collaboration of Wolfgang Pauli and Jung [4]. Despite the development of various human brain research, we still do not have a suitable set of mathematical languages to understand the consciousness [5].

We begin by introducing the recursive Heaviside step functions that the human psyche has no space to locate but exists only with time. The process of being conscious can be explained as constant process of differentiation of recursive Heaviside step function representing unconscious of human. Following Shin and Cha's approach of which our unconsciousness to a special solution of the multi-dimensional advection-like partial differential equation, we show that humankind is totally dependent upon unconscious that is represented by the infinite recursive Heaviside step function.

## 2. THE ADVECTION-LIKE EQUATION AND RECURSIVE HEAVISIDE STEP FUNCTION

The advection equation

$$\frac{\partial U}{\partial t} = -c \frac{\partial U}{\partial x} \quad (2.1)$$

governs the motion of an object that moves in one  $+x$  direction by velocity  $c$ , and the wave equation

$$\frac{\partial^2 U}{\partial t^2} = c^2 \frac{\partial^2 U}{\partial x^2} \quad (2.2)$$

represents the motion in both the  $+x$  and  $-x$  directions [6]. Because the human mental state proceeds only in a forward direction in time and should not depend on any particular space coordinates, we impose the advection-like equation

$$\frac{\partial U_n}{\partial t} = - \sum_{k=1}^n \frac{\partial U_n}{\partial \tau_k}, \quad (n = 1, 2, \dots) \quad (2.3)$$

to be satisfied by  $U_n$  which denotes the  $n$ -th particular situation pertaining to an individual person. In (2.3),  $\tau_k$ 's ( $k = 1, 2, \dots, n$ ) are virtual time variables that will be elaborated.

Previously, Shin and Cha proved by induction that the recursive Heaviside step function  $U_n(t, T_n)$  is a special solution of the advection-like equation [7], under the condition that the real time  $t$  is not larger than any one of the virtual time variables  $\tau_k$ s. The recursive Heaviside step function  $U_n(t, T_n)$  with depth  $n$  is defined in terms of the usual Heaviside step function  $H(-t + t_0)$  by [8]

$$U_n(t, T_n) = H[-t + \tau_n U_{n-1}(t, T_{n-1})] \quad (2.4)$$

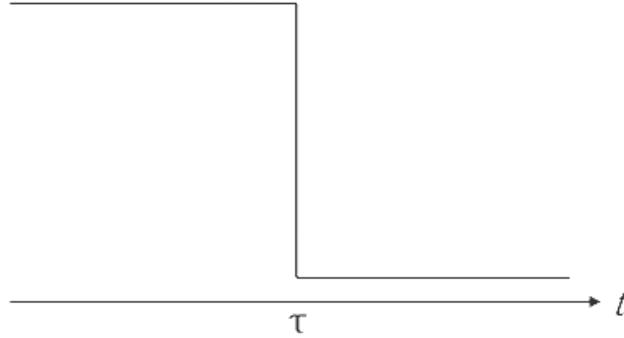


FIGURE 1. Heaviside step function  $H(-t + \tau)$ .

where

$$U_0(t, T_0) = 1, \quad H(-t + t_0) = \begin{cases} 1 & \text{when } t \leq t_0 \\ 0 & \text{when } t > t_0 \end{cases}$$

where  $T_n$  in (2.4) is a set of  $n$  virtual time variables  $T_n = (\tau_n, \tau_{n-1}, \dots, \tau_1)$  and  $T_0$  denotes the null.

Fig. 1. shows the shape of the Heaviside step function  $H(-t + \tau)$ . The shape of the recursive Heaviside step function  $U_n(t, T_n)$  is also same as the Heaviside step function shown above. The vertical axis means the value of the  $U_n$  which represents unconsciousness and the horizontal axis represent the time,  $\tau_n$  represent the present while  $\tau_{n-1}, \tau_{n-2}, \dots$  denote the past, and  $\tau_{n+1}, \tau_{n+2}, \dots$  denote the future.

### 3. UNCONCIOUS-TO-CONSCIOUS PROCESS

We postulate that the recursive Heaviside step function  $U_n(t, T_n)$  represents individual unconsciousness about a situation from which one aspect of a person’s mental state can be formed. This postulate is conceived because  $U_n(t, T_n)$  has an interesting property in which it gives a unique shape that is equal to that of the usual step function  $H(-t + \tau_0)$ , where  $\tau_0$  is the smallest among the virtual time variables  $\tau_k$  s, regardless of either the depth  $n$  or the values of the other  $\tau_k$  s.

We next demand that the partial derivative of the recursive Heaviside step function  $U_n(t, T_n)$  with respect to the real time  $t$  should be interpreted as consciousness of a person who experiences the situation represented by  $U_n(t, T_n)$ . Therefore, it can be said that the human brain in action is operating under a constant process of differentiation.

Figure 2 shows the typical example of recursive Heaviside step functions which represents the unconsciousness function of three persons. Jane, Paul and Bill are walking down the street together when a lost dog appeared in front of them. Jane shows no interest as she has had

Name	Mental State
Jane	 $U_1 = H(-t + \tau_1^J)$ $\frac{\partial U_1}{\partial t} = -\delta(-t + \tau_1^J)$
Paul	 $U_2 = H[-t + \tau_2^P H(-t + \tau_1^P)]$ $\frac{\partial U_2}{\partial t} = \delta[-t + \tau_2^P H(-t + \tau_1^P)] [-1 - \tau_2^P \delta(-t + \tau_1^P)]$
Bill	 $U_3 = H\{-t + \tau_3^B H[-t + \tau_2^B H(-t + \tau_1^B)]\}$ $\frac{\partial U_3}{\partial t} = \delta\{-t + \tau_3^B H[-t + \tau_2^B H(-t + \tau_1^B)]\}$ $\times \{-1 + \tau_3^B \delta[-t + \tau_2^B H(-t + \tau_1^B)] [-1 - \tau_2^B \delta(-t + \tau_1^B)]\}$

FIGURE 2. Unconsciousness function for Jane ( $U_1$ ), Paul ( $U_2$ ) and Bill ( $U_3$ ) with respect to same situation and consciousness for Jane, Paul and Bill represented by  $\partial U_1/\partial t$ ,  $\partial U_2/\partial t$ , and  $\partial U_3/\partial t$ , respectively.

no meaningful experience with a dog in the past. Paul, on the other hand, goes up to the dog and pats it on the head as he has spent a lot of time with dogs since childhood. However, Bill wants to stay away from the dog as far as he can, because he was once bitten by a dog when he was a child. Three people's conscious reaction to a lost dog may be different according to their unconsciousness structure cumulated from past experiences. Their unconsciousness of the situation is comparable to functions  $U_n$ ;  $U_1$  for Jane,  $U_2$  for Paul, and  $U_3$  for Bill. Three different reactions (=consciousness) can be expressed as time derivatives ( $\partial U_n(t, T_n)/\partial t$ ) of human unconsciousness using the recursive Heaviside step function.

#### 4. CONCLUSIONS

This article proves that the process of transformation of unconsciousness into consciousness can be expressed using the infinite recursive Heaviside step function and differentiation of recursive Heaviside step function with respect to current time. Differentiation of unconsciousness function  $U_n(t, T_n)$  with respect to time is the process of being conscious. The human reactions depend on past times. For every moment we live and experience, the unconscious constituents are accumulated. Then the human mind has heavy and innate overlapping of unconsciousness at times  $\tau_k$ . For instance, if humans become unconscious ten trillion different times, there are ten trillion squared different unconscious states including instances where various states co-exist. Consciousness derives from comparison of unconsciousness from the two consecutive moments.

Lives are sum of the integral and differential of every moments and so is the consciousness, which we can never summon into our mind again, even though we have realistically experienced. Single seconds and millimeters we experience can be divided forever, which the invention of scanning tunneling microscope shows. We need to redefine unconsciousness under the umbrella of modern mathematical theorem. Efforts to postulate the existence of unconsciousness through mathematical formulas in the form of recursive Heaviside step function may help AI to imitate, assist, and potentiate human mental activities. We ardently wait further research in the converging field of mathematics, physics, and psychiatry. Development of AI requires algorithmic regeneration of human mental activities on machines. Without numerical analysis of the unconscious potential energy, we can never build up any bridge between the limitless psychological and mechanical worlds.

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