Adaptive Methods for the Pseudostress Formulation of the Navier-Stokes Equations

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ABSTRACT

In this paper, multilevel algorithms are considered on abstract framework, which permits us to obtain approximate solutions of various partial differential equations (PDEs). We apply multilevel algorithms to stationary Navier-Stokes equations. It is well known that as the viscosity varies along an interval, each solution of the Navier-Stokes equations describes an isolated branch. This situation is expressed mathematically by the notion of branches of nonsingular solutions. Brezzi-Rappaz-Raviart (BRR) in [1] introduced and analyzed approximation of branches of nonsingular solutions for a class of nonlinear problem. BRR framework was utilized for the elliptic problem with gradient nonlinearities by Kim and Park [3].

Using different spaces for the domain and range of nonlinear operator in the application of BRR theory, we will obtain optimal a priori error estimates $O(h^{k+1})$ in the $L^3(\Omega)^d \times L^3(\Omega)^d$ norms. Also, exploiting multilevel algorithms as numerical algorithms that are able to provide approximate solutions, we derive error estimates bounded by $O(h^{k+1} + H^{2k+2})$ for coarse mesh size $H$ and fine mesh size $h$. Finally, residual based reliable and efficient a posteriori error estimators are derived for multilevel algorithms for the Navier-Stokes equations. The estimators for multilevel algorithms contains additional terms in comparison to a posteriori error estimators for standard finite element methods. Since the approximate solutions obtained by multilevel algorithms satisfy only violated error equations, the additional terms are inevitable.

REFERENCES