TIME-DOMAIN FULL-WAVEFORM INVERSION FOR PML-TRUNCATED SEMI-INFINITE DOMAINS

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ABSTRACT

We discuss a full-waveform inversion method for the reconstruction of the material profile of heterogeneous semi-infinite domains, directly in the time domain, based on scant surficial measurements of the domain’s response to prescribed wave illumination. Of particular interest is the ability to recover the in-depth profile of moduli/wave velocities typically associated with non-invasive condition assessment and structural health monitoring.

We address four key issues associated with the full-waveform inversion: a) to limit the semi-infinite extent of the physical domain, a perfectly-matched-layer (PML) is introduced at truncation interfaces to render a finite computational domain; b) to account for the introduction of the PML, while retaining the second-order temporal character of the hyperbolic problem, we discuss a new mixed unsplit-field formulation for the coupled PML-regular-domain problem; c) to tackle the inversion, we adopt a PDE-constrained optimization framework that formally leads to classic (time-dependent) KKT (Karush-Kuhn-Tucker) conditions; and d) to alleviate solution multiplicity, we discuss a first-order Tikhonov regularization scheme endowed with a regularization factor continuation algorithm. We report on one- and two-dimensional experiments that lead efficiently to the reconstruction of heterogeneous profiles involving both horizontal and inclined layers, as well as of inclusions within layered systems.

INVERSE PML FORMULATION

The inverse medium problem for reconstructing the shear wave velocity profile ($c_s$) in the PML-truncated domain can be cast as follows:

$$
\min_{c_s} J = \frac{1}{2} \int_{0}^{T} \int_{\Gamma_m} [v(\mathbf{x}, t) - v_m(\mathbf{x}, t)]^2 d\Gamma dt + R(c_s)
$$

subject to

$$
\nabla \cdot \left( \bar{\mathbf{F}}^e \ddot{s} + \bar{\mathbf{F}}^p \dot{s} \right) = f_m \frac{\partial^2 v}{\partial t^2} + c_s g_c \frac{\partial v}{\partial t} + c_s^2 g_k v
$$

$$
\mathbf{F}^e \ddot{s} + \mathbf{F}^p \dot{s} = c_s^2 \frac{\partial}{\partial t} (\nabla v)
$$

$$
v(\mathbf{x}, t) = 0, \quad \text{for } \mathbf{x} \in \Gamma_{\text{fixed}}, t \in [0, T]
$$

$$
\frac{\partial s_2}{\partial t} (\mathbf{x}, t) = g(\mathbf{x}, t), \quad \text{for } \mathbf{x} \in \Gamma_{\text{free}}, t \in [0, T]
$$
\[ \begin{align*}
v(x, 0) &= \frac{\partial v}{\partial t}(x, 0) = 0, \quad \text{for } x \in \Omega_{\text{reg}} \cup \Omega_{\text{PML}} \quad (3c) \\
s(x, 0) &= \dot{s}(x, 0) = 0, \quad \text{for } x \in \Omega_{\text{reg}} \cup \Omega_{\text{PML}} \quad (3d)
\end{align*} \]

To invert for the shear wave velocity profile, a PDE-constrained optimization method was implemented, which seeks the stationarity of the augmented Lagrangian functional by the first-order optimality conditions. The approach formally leads to classic KKT conditions characterized by initial-value state, final-value adjoint, and time-independent control problems. The KKT conditions were iteratively resolved to lead to the reconstruction of the desired shear wave velocity profile.

**NUMERICAL EXAMPLES**

We report on the reconstruction of highly heterogeneous Marmousi benchmark velocity profile using the proposed inversion method, as shown in Fig. 1.

![Figure 1 Reconstruction of the Marmousi velocity profile](image)

(a) Target profile  
(b) Reconstructed profile

**REFERENCES**