Strong edge-colorings of sparse graphs

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ABSTRACT

A strong k-edge-coloring of a graph $G$ is a mapping from $E(G)$ to $\{1, 2, \ldots, k\}$ such that every two adjacent edges or two edges adjacent to the same edge receive distinct colors. The strong chromatic index $\chi'_s(G)$ of a graph $G$ is the smallest integer $k$ such that $G$ admits a strong $k$-edge-coloring. We give bounds on $\chi'_s(G)$ in terms of the maximum degree $\Delta(G)$ of a graph $G$ when $G$ is sparse, namely, when $G$ is 2-degenerate or when the maximum average degree $\text{Mad}(G)$ is small. We prove that the strong chromatic index of each 2-degenerate graph $G$ is at most $5\Delta(G) + 1$. Furthermore, we show that for a graph $G$, if $\text{Mad}(G) < 8/3$ and $\Delta(G) \geq 9$, then $\chi'_s(G) \leq 3\Delta(G) - 3$ (the bound $3\Delta(G) - 3$ is sharp) and if $\text{Mad}(G) < 3$ and $\Delta(G) \geq 7$, then $\chi'_s(G) \leq 3\Delta(G)$ (the restriction $\text{Mad}(G) < 3$ is sharp).