AN IMMERSED INTERFACE METHOD FOR SOLVING THE TWO-FLUID NAVIER-STOKES EQUATIONS

Zhijun Tan [1] , D.V. Le [1] , K.M. Lim [2] and B.C. Khoo [2]

1) *Singapore-MIT Alliance Mechanical Engineering, E1-02-01 Dynamics Lab, 10 Kent Ridge Crescent, National University of Singapore, Singapore 119260*

2) *Department of Mechanical Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260*

Corresponding Author : Zhijun Tan, smatz@nus.edu.sg

ABSTRACT

In this work, we present an immersed interface method for solving the incompressible Navier-Stokes equations with discontinuous viscosity across the interface and singular forces. The method combines the augmented immersed interface method with front tracking representation of the interface on a uniform Cartesian grid. The immersed interface is represented by a number of Lagrangian control points, and the augmented strategy is based on the approach proposed by Li et al. [*Comput. Fluids.*, 36 (2007), pp. 622–635] to decouple the jump conditions of the fluid variables through two augmented variables. In the proposed method, the augmented interface variables are determined by solving a small system of equations by the LU method or GMRES iterative method. The forces, the augmented variables and their derivatives along the interface, which are related to the jumps in pressure and the jumps in the derivatives of both pressure and velocity, are interpolated using cubic splines. For flexible interface, the forces that the interface exerts on the fluid are computed from the constitutive relation of the flexible interface and are applied to the fluid through the jump conditions. The position of the flexible interface is updated implicitly using a quasi-Newton method (BFGS) within each time step. The Navier-Stokes equations are discretized on a staggered Cartesian grid by a second order accurate projection method, and a modified bilinear interpolating scheme for the non-smooth even discontinuous velocity has been developed. The numerical results show that the overall scheme is second order accurate.

REFERENCES

1. Z. Li, K. Ito and M-C. Lai, “An augmented approach for Stokes equations with a discontinuous viscosity and singular forces”, *Computers and Fluids*, Vol. 36, 2007, pp. 622-635.
2. Z. Li and M-C. Lai, “The immersed interface method for the Navier-Stokes equations with singular forces”, *Journal of Computational Physics*, Vol. 171, 2001, pp. 822-842.
3. D.V. Le, B.C. Khoo and J. Peraire, “An immersed interface method for viscous incompressible flows involving rigid and flexible boundaries”, *Journal of Computational Physics*, Vol. 220, 2006, pp. 109-138.

4. R.J. LeVeque and Z. Li, "The immersed interface method for elliptic equations with discontinuous coefficients and singular sources", *SIAM Journal on numerical Analysis*, Vol. 31, 1994, pp. 1019-1044.
5. A. Wiegmann and K.P. Bube, "The explicit-jump immersed interface method: finite difference methods for PDEs with piecewise smooth solutions", *SIAM Journal on numerical Analysis*, Vol. 37, pp. 827-862.
6. S. Xu and Z. J. Wang, "Systematic derivation of jump conditions for the immersed interface method in three-dimensional flow simulation", *SIAM Journal on Scientific Computing*, Vol. 27, 2006, pp. 1948-1980.
7. S. Xu and Z. J. Wang, "An immersed interface method for simulating the interaction of a fluid with moving boundaries", *Journal of Computational Physics*, Vol. 216, 2006, pp. 454-493.