Computational Models of Valveless Pumping Using the Immersed Boundary Method

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Abstract

Mathematical models of valveless pumping can be represented by either a closed loop system or an open tube system. In this paper, we present a three-dimensional model of valveless pumping in a closed loop system. We also present a two-dimensional model using an open elastic cylinder contained in a rigid tank. In both models, we take the periodic compress-and-release action at the asymmetric location of the soft tube and observe the existence of a net flow and the important features of valveless pumping that have been reported in the previous models or experiments. The innovative idea of this work is that we explain the existence of a net flow by introducing the concept of the signed area of the flow-pressure loop over one cycle, which represents the power in the system. The direction and the magnitude of a net flow can also be explained by the sign and the amount of power, which is work done on the fluid by the fluid pressure and the elastic wall over one period, respectively.

1. Introduction

Mathematical models of valveless pumping can be represented by either a closed loop system or an open system that consists of a couple of tubes with different elasticities or radii. A unidirectional net flow is observed in a valveless pump model when the asymmetric periodic force is applied on the system. We call this phenomenon *valveless pumping*. For decades, both experimental and computational researches on valveless pumping have been intensively studied. Research on valveless pumping was originally motivated to understand the mechanism of the blood circulatory system. There are cases in nature of unidirectional net flow in the circulatory system even with malfunctioning valves. For example, the mechanism of valveless pumping may explain unidirectional blood circulation in the human embryo prior to the end of the third week of gestation, when cardiac valves first are developed [18]. Scientists proposed theories to explain the mechanism of a unidirectional blood flow during cardiopulmonary resuscitation (CPR). One of the main theories is a thoracic pump mechanism, in which blood circulation occurs when some of the valves do not function normally [1, 2, 3, 4]. Valveless pumping may explain the underlying mechanism of the thoracic pump. Recently, the physical experiments of a micro-scaled

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version of valveless pumping in an open or a closed loop system were developed by Hickerson *et al.* [5, 22]. The mechanism of valveless pumping can be applied not only to macrofluid dynamics but also to microfluidics and micro devices.

A physical experiment of valveless pumping in a closed loop model was first suggested by Liebau [9, 10, 11]. Liebau presented the mechanism of valveless pumping in the circulation during early embryonic life [9], energy transfer from arterioles to venules [12], and the tissue capillary circulation [13]. Thomann initiated a mathematical model, a one-dimensional model for periodic, inviscid, and incompressible flow [23]. Moser *et al.* presented a lumped electric model in a closed loop [19]. Jung and Peskin first developed a two-dimensional computational model of valveless pumping in a closed loop [6, 7]. They showed the direction and magnitude of a net flow are dependent not only on the position of pumping but also parameters such as frequency of the oscillatory force. Ottesen developed a one-dimensional model of valveless pumping by averaging the Navier-Stokes equations, ignoring higher-order terms in a certain small quantity [20]. Recently, Manopoulous *et al.* have reported qualitative results from their quasi-one-dimensional model [15]. Jung developed a simple lumped model system governed by the ODE equations with the time-dependent compliances, resistances, and inertia [8].

In this paper, we present two new models of valveless pumping using the immersed boundary method. A three-dimensional model of valveless pumping in a closed loop system is first presented. As mentioned earlier, the various types of the mathematical models of valveless pumping have been developed, for example, lumped models [8, 19], one-dimensional models [15, 20, 23], and two-dimensional models [6, 7]. These models have provided qualitative analysis and discovered new mechanisms of valveless pumping. However, the details of fluid motions or the wave motions along the tube can not be investigated in the previous models. The three-dimensional model will allow us to analyze the fluid dynamics in a valveless pump.

Next, we present a two-dimensional model of valveless pumping. The two-dimensional model is composed of an open elastic tube enclosed by a rigid rectangle, and they are separated unlike most other related approaches: *two different materials do not need to be connected in a valveless pump system to generate a net flow*. Although the elastic and the rigid parts are separated, we have observed main features of valveless pumping. A striking result of this work is that we could explain the fundamental question of valveless pumping: why does a net flow exist in a valveless pump system? We introduce the concept of the signed area of a flow-pressure loop over one cycle, which implies the power. With the concept of the net signed area, the direction and magnitude of a net flow can be explained in terms of the sign and the amount of power, which is work done on the fluid by the fluid pressure and the elastic wall over one period, respectively. In the two-dimensional and three-dimensional models, we have showed that the direction and magnitude of the net flow are dependent not only upon the position of pumping but also the frequency and the amplitude of the driving function.

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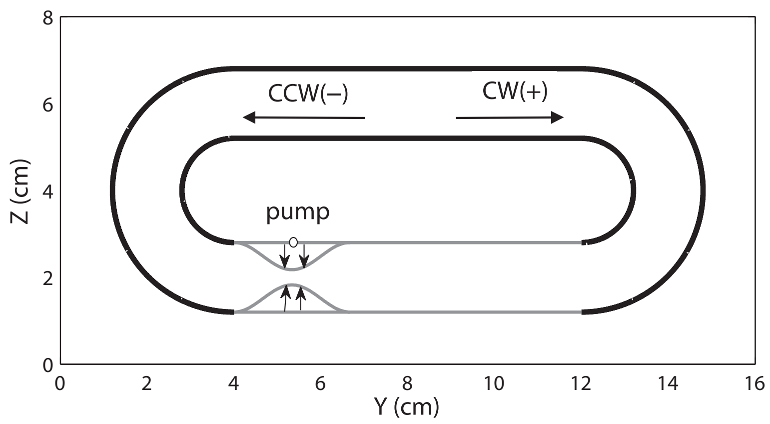

Figure 1: A cross section in the middle of a three-dimensional model of valveless pump immersed in a viscous fluid. There are two flow directions: counterclockwise (CCW) and clockwise (CW). The closed loop consists of a rigid tube (thick line) and a soft tube (thin line). Periodic pumping force is applied to one third of the soft tube from the left end.

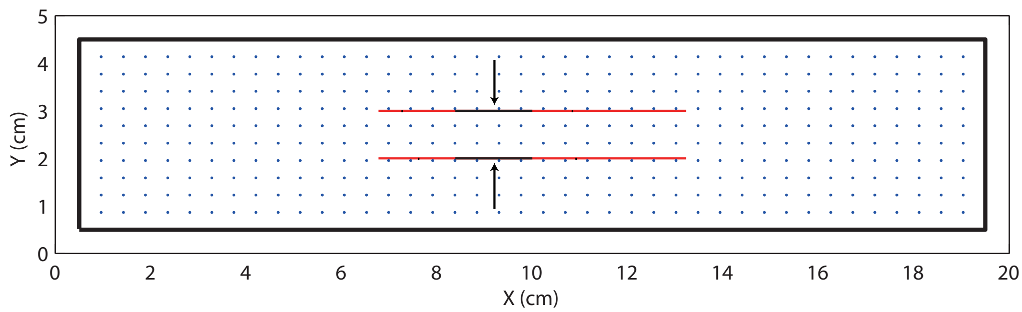

Figure 2: Initial configuration: Outer rectangular box represents a computational domain. A closed rigid tank (thick line, inner rectangular box), an open elastic tube (thin lines in red), and fluid markers (dots) are displayed. The periodic pumping is applied to the left second quarter of the elastic tube (arrows).

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