FINITE ELEMENT CHARACTERISTIC METHODS FOR FLOW PROBLEMS

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ABSTRACT

A remarkable feature of flow problems compared with others is that there exists the material derivative term

$$\frac{D\phi}{Dt} \equiv \frac{\partial \phi}{\partial t} + u \cdot \nabla \phi \equiv \frac{\partial \phi}{\partial t} + \sum_{j=1}^{d} u_j \frac{\partial \phi}{\partial x_j},$$

where u is a function depicting the flow field and ϕ is unknown physical quantity such as the density, the velocity, or the energy. It makes problems asymmetric in any time and nonlinear when the velocity field is unknown, e.g., in the Navier-Stokes equations ϕ stands for each component u_i of unknown velocity u, which leads to the nonlinear term $u \cdot \nabla u_i$. The combination of this term with the diffusion term $-\nu \Delta \phi$ describes many important phenomena in sciences and engineering. It produces various fruitful and interesting results, especially when the diffusion constant ν is small, e.g., high Reynolds number problems in the Navier-Stokes equations. In devising numerical schemes for the solution of those phenomena, it is well-known that the discretization of this term is crucial because the conventional Galerkin finite element method and the centered difference method easily produce unphysical oscillating solutions. In some remedies for the instability the method of characteristics seems to be natural from the physical point of view since it approximates the particle movement along the trajectory. Here we consider finite element methods based on characteristics; they are called by some different names, e.g., Lagrange-Galerkin method, Eulerian-Lagrangian method, characteristics/finite element methods and so on. Those methods lead to symmetric schemes which are robust even for convection-dominated problems. After reviewing the first order method in time increment, we discuss finite element characteristic methods on the following issues;

- second-order approximation in time increment,
- mass-conservation approximation,
- some relating topics.

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