Data-Driven Scientific Computing

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ABSTRACT

Scientific computing is an important tool in solving complex problems in sciences and engineering. In many real world applications, we are interested in the study of nonlinear physical problems. For obvious reasons related to the safety issue, it is important to develop methods which are capable of forecasting the nonlinear response of a physical system. Knowing the possibility of encountering unstable behavior before they occur can help to prevent catastrophic consequences.

Traditionally, given a practical problem, we carry out analysis and derive a mathematical model. The solution of the governing differential equations with boundary and initial conditions are then computed using numerical techniques. In recent years, there has been a growing interest in utilizing and integrating data in dealing with many difficult engineering applications. In this talk, we present a data-driven scientific computing approach, and we demonstrate that the method is capable of predicting the asymptotic behavior of the nonlinear aeroelastic system from a limited transient data.

A two degree-of-freedom (DOF) aeroelastic system simulating an oscillating airfoil in pitch and plunge can be described by the following equations:

$$\xi'' + x_\alpha \alpha'' + 2\zeta_\xi \frac{\tilde{\omega}}{U^*}\xi' + \left(\frac{\tilde{\omega}}{U^*}\right)^2 G(\xi) = -\frac{1}{\pi\mu} C_L(t) \tag{1}$$

$$\frac{x_\alpha}{r_\alpha^2}\xi'' + \alpha'' + 2\frac{\zeta_\alpha}{U^*}\alpha' + \frac{1}{U^{*2}} M(\alpha) = \frac{2}{\pi\mu r_\alpha^2} C_M(t), \tag{2}$$

where x_α, r_α, ζ_α, ζ_ξ, $\tilde{\omega}$, U^*, μ are the airfoil parameters, and they are defined in Ref. [2]. Here $G(\xi)$ and $M(\alpha)$ are the nonlinear plunge and pitch stiffness terms,

respectively. $C_L(t)$, $C_M(t)$ are the lift and pitching moment coefficients, and they are expressed by integral terms for subsonic flows. The above aeroelastic system is described by two integro-differential equations, but it can be rewritten as a system of eight nonlinear ordinary differentiating equations.

In general, non-linear behavior in aeroelasticity arises either from the aerodynamics or from the structure. In transonic flow regimes, the non-linearities in aerodynamics cannot be ignored due to the presence of shock oscillations. However, in low speed subsonic regimes, linear aerodynamics is usually assumed. In the structure, non-linearities may occur in the restoring forces, and can be classified as polynomial springs or piece-wise linear types, such as freeplay or hysteresis. Freeplay non-linearity can occur in the control surfaces or components with loose joints. It has been observed that even a small amount of freeplay could lead to limit cycle oscillations (LCOs) [1].

In developing a data-driven method, we start with a time series of measured data. In Fig. 1, samples of transient aeroelastic data are displayed. Due to the space constraint, only one of the time-histories of the pitch angle or the plunge displacement is shown. However, in actual applications, both the pitch and plunge data are employed. The time series illustrated in Fig. 1 have not reached the steady-state, and they are generated by experiments or numerical simulations. Obviously, it is difficult to forecast the asymptotic response based on the limited data. In each case, the long-term behavior could be a limit-cycle-oscillation, non-oscillating fixed-point solution, divergent, or even a chaotic motion.

Using a limited transient data as input, we first apply statistical techniques to identify the specific type of the nonlinearity associated with the aeroelastic model. Knowing the type of the nonlinearity enable us to propose an appropriate mathematical model which exhibit the behavior consistent with the measured data. By the application of the Kalman filter and the expectation maximization algorithm, the unknown system parameters are estimated. Consequently, the nonlinear system response can be predicted from the reconstructed model.

Results using the developed data-driven tool applied to aeroelastic data resulting from experiment investigations and from numerical simulations are reported to demonstrate the effectiveness of this technique. It is worth noting that in addition to using the system identification method presented here, artificial neural networks [4]and nonlinear time-series models [3] have also been incorporated in developing a data-driven scientific computing tool.

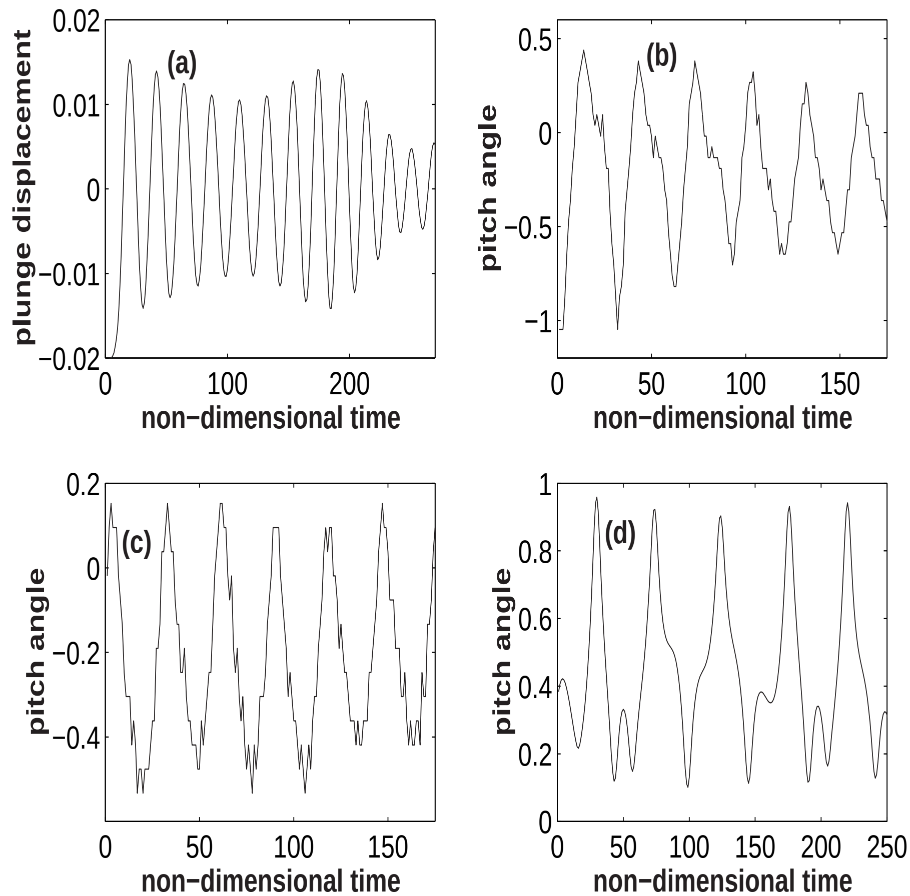

Figure 1: Samples of input pitch data for data-driven method

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