Some remarks on the Aubin-Nitsche trick of FEM solutions for elliptic problems with singular adjoint operator

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ABSTRACT

Consider a weak solution u and a finite element solution u_h for the linear elliptic equation of the form

$$-\triangle u + b \cdot \nabla u + cu = f \quad in \;\; \Omega,$$
$$u = 0 \quad on \;\; \partial\Omega.$$

Here, $\Omega \subset \mathbb{R}^d$ $(d = 1, 2, 3)$ is a convex polygonal (polyhedral) domain, and b, c and f are given functions in $L^\infty(\Omega)^d,\ L^\infty(\Omega)$ and $L^2(\Omega)$, respectively. Then, if the coefficient function b is smooth, the a priori error estimates $||u - u_h||_{L^2(\Omega)}$, which is one order higher than H_0^1 error, can be usually obtained by so called *Aubin-Nitsche's trick*. However, if b is discontinuous, such a technique, could no longer be applied, because the solution of the dual problem has no regularity at all. In such a nonsmooth case, it seems that there are no theoretical results, up to now, on the higher order L^2 error estimates than H_0^1 error. In this talk, we present a method to get L^2 a priori error estimates by using a self-validating numerical computations. We will show some computational results on the a priori L^2 error estimates with almost optimal order by using a new formulation combining with a guaranteed numerical computation for solutions of some kind of saddle point problem. These results imply a mathematically rigorous proof and are considered as a computer assisted proof in error analysis for FEM.

REFERENCES

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