An Algebraic Multigrid Preconditioner for Solving the Transient Navier-Stokes Equation on Adaptive Meshes

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Multigrid methods are very popular in large scale computation in both science and engineering communities recently. On the other hand, adaptive error controls are devised to increase accuracy of numerical solutions without exponentially increasing the computation cost. In this paper, we solve the transient Navier-Stokes equation (TNS) for incompressible flows on adaptive meshes where the adaptive meshes are generated by applying the Kay and Sylvester a posteriori error indicator to the vorticity formulation of the TNS equation. The TNS equation is then discretized by using Crank-Nicolson method for time domain and the streamline diffusion finite element method (SDFEM) with P1/P1 elements for the spatial domain. The resulting discrete system can be represented as follows:

$$\begin{bmatrix} F_{11} & F_{12} & B_x \\ F_{21} & F_{22} & B_y \\ \tilde{B}_x & \tilde{B}_y & C \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \\ p \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ 0 \end{pmatrix} .$$

We solve the above system by a block-preconditioned general minimized residual method (GMRES) in which the block-preconditioner has the following form

$\begin{bmatrix} \bar{F}_{11} & 0 & 0 \\ 0 & \bar{F}_{22} & 0 \\ \tilde{B}_x & \tilde{B}_y & C \end{bmatrix}$, where $\bar{F}_{11}$, $\bar{F}_{22}$ and C are essentially the stabilized Hemholtz operators and Poisson operator, respectively. Furthermore, instead of solving each block directly, one step of the v-cycle of the algebraic multigrid method (AMG), proposed by Ruge and Stüben, is employed for each block solve in order to save the computation time. This preconditioner is shown to be very efficient in several benchmark problems including flows passing a cylinder or an airfoil, and flows in a driven cavity.