

Numerical Method for Maxwell Equation by FDTD Method with PML Applied to MRI Problem

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ABSTRACT

In the numerical simulation of electromagnetic wave propagation phenomena in unbounded region, it is necessary to set a computational bounded domain.

In 1994, Berenger [1] introduced the so-called perfectly matched layer (PML in short) that surrounds a bounded vacuum region. In the PML region, there is no reflection for the wave coming from the vacuum region and hence there is no change of the solution in the bounded vacuum region. The PML is defined as the region consisting of the imaginary dissipative material. By this method combined with some artificial splitting of the physical variable, the two and three-dimensional electromagnetic wave phenomena can be simulated efficiently. As for the discretization, the PML technique combined with the Finite Difference Time Domain (FDTD) method (see [3]) gives a good numerical performance.

In this paper, we consider the numerical simulation of two and three dimensional electromagnetic wave propagation in Magnetic Resonance Imaging (MRI in short) device with bird-cage shape antennas surrounding the target object that is modeled by the region with non-zero conductivity. We compute a space-time wave profile for the case with the antenna emitting the radio frequency wave. In application, it is important to compute Specific Absorption Rate (SAR in short) to estimate the client's safety under the investigation by MRI device.

We explain the basic idea of PML is as follows (see [2]). Let two unknown functions $u(t, x)$ and $v(t, x)$ satisfy the equation

$$\frac{\partial u}{\partial t}(t, x) = -\frac{\partial v}{\partial x}(t, x), \quad \frac{\partial v}{\partial t}(t, x) = -\frac{\partial u}{\partial x}(t, x). \quad (1)$$

We consider the initial value problem of this equation in whole one-dimensional space R . The exact solution for the initial value problem of this equation is explicitly written as

$$u(t, x) = f(x - t) + g(x + t), \quad v(t, x) = f(x - t) - g(t + x) \quad (2)$$

with functions f and g defined as

$$f(x) = \frac{1}{2} \left(u(0, x) + v(0, x) \right), \quad g(x) = \frac{1}{2} \left(u(0, x) - v(0, x) \right). \quad (3)$$

Now, we introduce a dissipation term $\sigma(x)u(t, x)$ in the left-hand side of this equation :

$$\frac{\partial u}{\partial t}(t, x) + \sigma(x)u(t, x) = -\frac{\partial v}{\partial x}(t, x), \quad (4)$$

$$\frac{\partial v}{\partial t}(t, x) + \sigma(x)v(t, x) = -\frac{\partial u}{\partial x}(t, x). \quad (5)$$

We assume that the dissipation term $\sigma(x)$ is a nonnegative real valued locally integrable function of x . In practice, we take σ to be a piecewise polynomial having possible discontinuities. The solution of this modified wave equation with dissipation term is given exactly as

$$u(t, x) = \frac{1}{2} \left(e^{-\int_0^x \sigma(s) ds} f(x-t) + e^{\int_0^x \sigma(s) ds} g(x+t) \right),$$

$$v(t, x) = \frac{1}{2} \left(e^{-\int_0^x \sigma(s) ds} f(x-t) - e^{\int_0^x \sigma(s) ds} g(x+t) \right),$$

with

$$f(x) = \frac{1}{2} e^{\int_0^x \sigma(s) ds} \left(u(0, x) + v(0, x) \right), \quad g(x) = \frac{1}{2} e^{-\int_0^x \sigma(s) ds} \left(u(0, x) - v(0, x) \right).$$

Hence, if $\sigma(x) = 0$ when $-L < x < L$ and $|\sigma(x)| > \sigma_0 > 0$, outside some bounded region, the solutions u and v coincide with those for the original equation when $-L < x < L$ and they attenuate exponentially as $|x|$ tends to infinity for such initial values u and v that are given through uniformly bounded functions f and g . The region with $\sigma(x) \neq 0$ is called perfectly matched layer (PML) according to the idea of Berenger for its introduction.

We can extend this formulation at least formally to two and three-dimensional cases using another idea of Berenger to split each physical variable into two corresponding computational ones for introducing PML with different artificial conductivity σ .

Related to the source term of electromagnetic wave, we consider two kinds of boundary condition on the antennas and solve the radiation problem as the mixed initial boundary value problem. We compare numerically the wave propagation profiles given by those different formulations.

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