A UNIFIED APPROACH TO POPULAR COMPUTATIONAL METHODS FOR SPDES

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Physical, biological, social, economic, financial, etc. processes always involve uncertainties. *Uncertainty quantification* is the task of determining statistical information about outputs of a system or process, given statistical information about the inputs. Here, we are interested in systems for which solutions of a partial differential equation (PDE) are used to define the mapping from the input variables to the output variables. Accounting for uncertainty in processes governed by PDEs can involve input data in the form of random coefficients and right-hand sides in the PDEs, boundary conditions, and initial conditions and even random geometries, i.e., random boundary shapes. Uncertainty arises because available data are incomplete or unpredictable.

We are interested in stochastic PDE (SPDE) problems whose definition involves a finite number which is independent of the spatial grid size, of random parameters. The parameters may directly define the input data, e.g., “knobs” in an experiment, or may result from, e.g., Karhunen-Loevy, approximations of correlated random fields.

We review, compare, and contrast several popular methods for determining output quantities of interest that depend on solutions of SPDEs. The main objective is to explore the connections between the different methods. In particular, we will show how all these methods can be put into a single framework, i.e., they are all special cases of *stochastic Galerkin methods.* This is not merely a mathematical nicety; placing the methods within a single framework facilitates making connections between them and comparing their relative merits. The discussion includes, but is not limited to, intrusive and non-intrusive polynomial chaos, stochastic collocation, and stochastic sampling methods.