On Vector Matrix Game and Vector Duality

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ABSTRACT

A matrix game is defined by B of real $m \times n$ matrix together with the Cartesian product $S_n \times S_m$ of all m-dimensional probability vectors S_n and all n-dimensional probability vectors S_m, that is, $S_n := \{x = (x_1, \cdots, x_n) \in \mathbb{R}^n : x_i \geqq 0, \sum_{i=1}^n x_i = 1\}$ and $S_m := \{x = (x_1, \cdots, x_m) \in \mathbb{R}^m : x_i \geqq 0, \sum_{i=1}^m x_i = 1\}$. The symbol ^{T} denotes the transpose.

A point $(\bar{x}, \bar{y})$ in $S_n \times S_m$ is a solution(equilibrium point) of the game B if $x^T B\bar{y} \leqq \bar{x}^T B\bar{y} \leqq \bar{x}^T By$ for all $x \in S_n$ and for all $y \in S_m$ and $\bar{x}B\bar{y} = v$, where v is value of the game.

We let $\overset{o}{S}_n = riS_n$, where riS_n is the relative interior of S_n.

Consider the linear programming problem (LP) together with its dual (LD) as follows:

(LP) Minimize $c^T x$, subject to $Ax \geqq b,\ x \geqq 0$,

(LD) Maximize $b^T y$, subject to $A^T y \leqq c,\ y \geqq 0$,

where $c \in \mathbb{R}^n, x \in \mathbb{R}^n, b \in \mathbb{R}^m, y \in \mathbb{R}^m, A = [a_{ij}]$ is an $m \times n$ real matrix.

Now consider the matrix game associated with the following $(n+m+1) \times (n+m+1)$ skew symmetric matrix B related to (LP) and (LD):

$$B = \begin{bmatrix} 0 & A^T & -c \\ -A & 0 & b \\ c^T & -b^T & 0 \end{bmatrix}.$$

The following theorems are due to Dantzig ([3]) are well known ([5], [6]): Theorem 1.1 and 1.2 give complete equivalence between linear programming duality and the matrix game B.

Theorem 1.1. *Let $\bar{x}$ and $\bar{y}$ be optimal solutions to (LP) and (LD) respectively. Let $z^* = 1/(1 + \sum_j \bar{x}_j + \sum_i \bar{y}_i),\ x^* = z^*\bar{x},\ y^* = z^*\bar{y}$. Then (x^*, y^*, z^*) solves the matrix game B.*

Theorem 1.2. *Let (x^*, y^*, z^*) be an optimal strategy of the matrix game B with $z^* > 0$. Let $\bar{x}_j = (x_j^*/z^*),\ \bar{y}_i = (y_i^*/z^*)$. Then $\bar{x}$ and $\bar{y}$ are optimal solutions to (LP) and (LD) respectively.*

Many authors [1, 2, 4, 7, 8] have extended Theorems 1.1 and 1.2 to several kinds of (scalar) optimization problems. The pupose of this paper is to extend Theorems 1.1 and 1.2 to a linear vector optimization problem which minimize simultaneously more than two linear objective functions over a polyhedral convex set. A vector matrix game with more than two skew symmetric matrices, which is an extension of the matrix game, is defined. We formulate a Wolfe

type dual problem for a linear vector optimization problem, give a weak duality result for the dual problem and establish equivalent relations between the dual problem and the vector matrix game. Moreover, we give numerical examples for vector matrix games and the equivalent relations.

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