FEM on Nonlinear Free Surface Flow

Kwang June BAI[1]

1) *Department of Naval Architecture and Ocean Engineering, Seoul National University, Seoul 151-744,Korea*

Corresponding Author: Kwang June BAI, kjbai@snu.ac.kr

ABSTRACT

Recently there has been a growing need for the treatment of the nonlinear water wave problems. In this paper we will present an overview on the finite element method applied to the nonlinear water wave problems, i.e., in ship motions, wave resistance, lifting problems, and initial value problems. The present numerical scheme can be used for the validation of the existing approximate theories, i.e., the KdV and the Boussinesq equations or Green-Naghdi equation as well as for a better prediction for more realistic physical models.

In the paper we discuss the following applications: The finite element method applied to a two dimensional hydrofoil problem. An initial-value problem for a physical model of an axi-symmetric container filled with water freely falling into a flat solid surface. The generation of solitons in shallow water towing-tank near the critical speed as well as a numerical towing tank simulation for arbitrary tank conditions. A sloshing and a diffraction problems. Finally, a brief concluding remark is given. A full account of this paper can be found in [1]

NONLINEAR STEADY WAVES DUE TO A TWO-DIMENSIONAL HYDROFOIL

An application is described of the localized finite-element method to a steady nonlinear free-surface flow past a submerged two-dimensional hydrofoil at an arbitrary angle of attack. It should be noted that in the earlier treatment of the linear hydrofoil problems, the velocity potential formulation was used. However, in the following treatment of the nonlinear problem, a stream function formulation is used since it has a distinct advantage in the numerical treatment. A detail can be found in [2,3]

Variational principle

The Lagrangian L for the above nonlinear boundary value problem can be defined as

$$L[\psi,\eta] = \frac{1}{2}\iint_D \nabla\psi\cdot\nabla\psi \, dxdy - \frac{g}{2}\int_{\overline{S_F}} \eta^2 dx \qquad (1)$$

with the kinematic constraints.

Numerical results

The computations are made for several values of Froude numbers and the depths of submergence. Figures 3.4 show the result for Froude numbers 0.989 with a depth of submergence, h/c = 0.2.

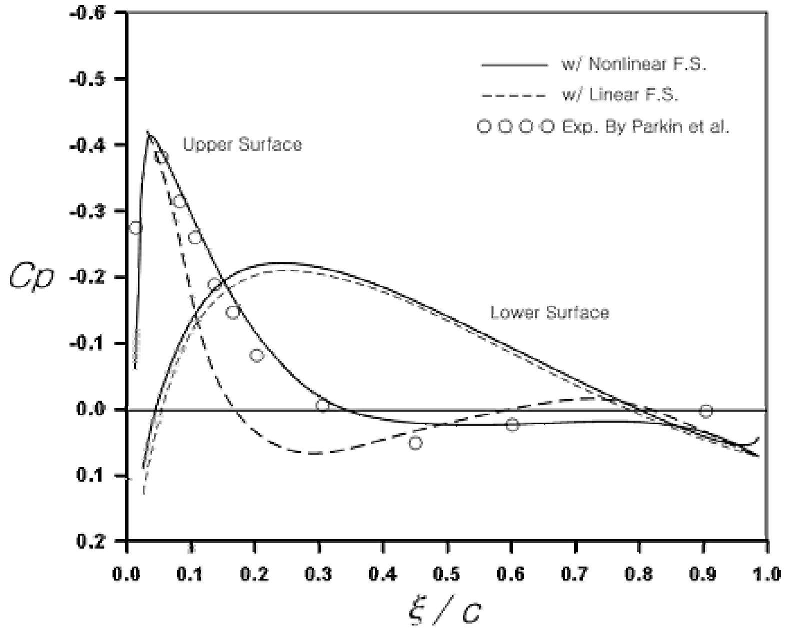


Figure 1. Pressure distribution ($F_c = 0.989,\ H/c = 6, h/c = 0.2$)

AXI-SYMMETRIC TRANSIENT PROBLEM

Here the time dependent motion of a fluid in a vertical circular cylindrical container with a free surface subject an impact is discussed.. The free surface abruptly rises very high just after this impact. Therefore it seems to be necessary to apply the nonlinear free surface condition for the problem. This problem was originally investigated experimentally by Milgram in [4], who performed a series of experiments and linear analysis for a vertical impact due to the collision between a free falling container filled with a fluid and the ground. The fluid is assumed to be inviscid and incompressible with its motion being irrotational. However, the surface tension is included since it plays an essential role in unique restoring force during the free fall of the container. A detail account can be found in [1].

Physical Model and Assumptions

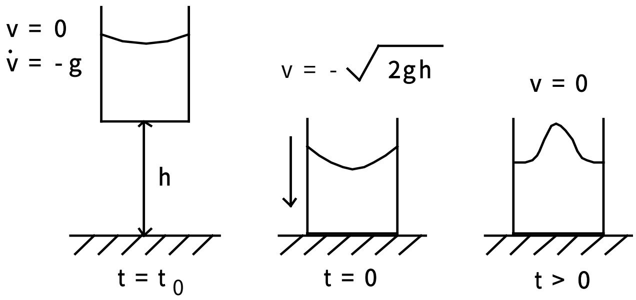
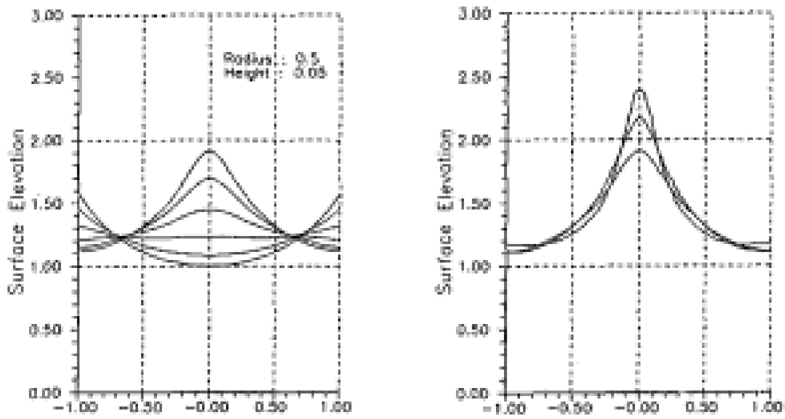


Figure 2. Perfect plastic collision of the container

Numerical Results



(a) t=0.0, 0.1,···, 1.0 (b) t=1.0, 0.3, 1.9

Figure 3. Free-surface elevation

NUMERICAL TOWING TANK SIMULATION

Here we describe a finite element method applied to a nonlinear free surface flow problem for a ship moving in three dimensions given in [5,6].

Variational Formulation

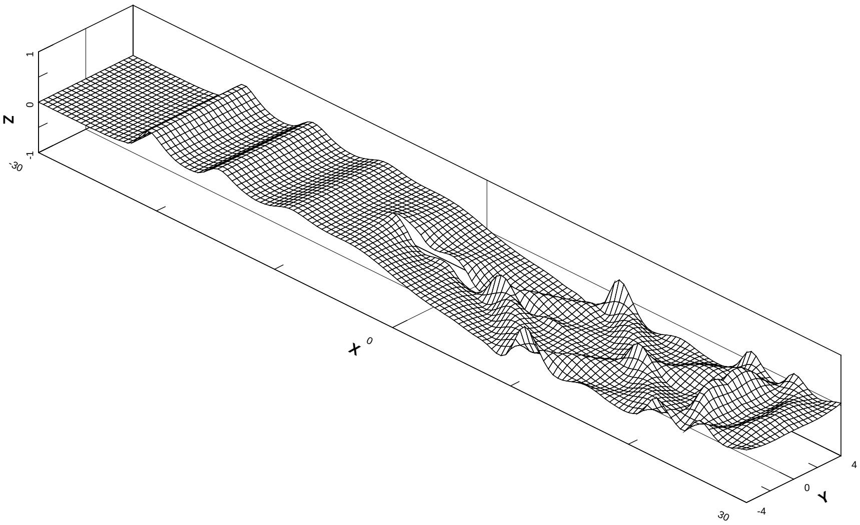
We introduce a variational formulation which is equivalent to the above problem. First we define the variational functional, J and the Lagrangian L as

$$J = \int_0^{t^*} L dt \tag{2}$$

$$L = \iint_{\bar{S}_F} \phi \zeta_t dS - \frac{1}{2} \iint_{\bar{S}_F} \zeta^2 dS - \frac{1}{2} \iiint_D |\nabla \phi|^2 dV \tag{3}$$

Where $\bar{S}_F$ is the projection of S_F on Oxy plane and t^* is the final time as given in [7,8].

The model is a vertical wall-sided wedge-shaped bow and stern extended from the free surface to the bottom of the tank.



After 100 sec

Figure 4. Computed free surface elevation for $F_h = 1.0$

NONLINEAR SLOSHING AND DIFFRACTION PROBLEMS

Here a nonlinear sloshing problem in LNG tanker in [9]and a diffraction problem in [10] in Stokes wave are numerically simulated.

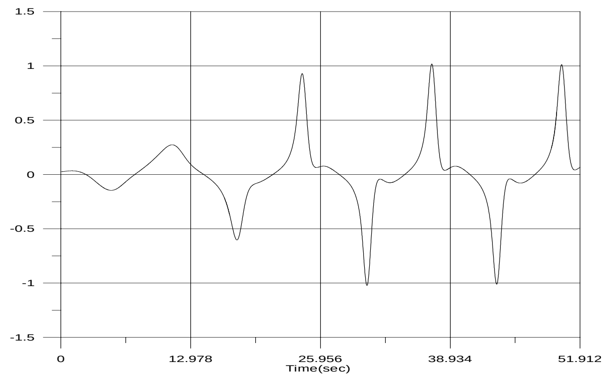


Figure 5. Impact force on the pillar versus time*: h= 4 m*

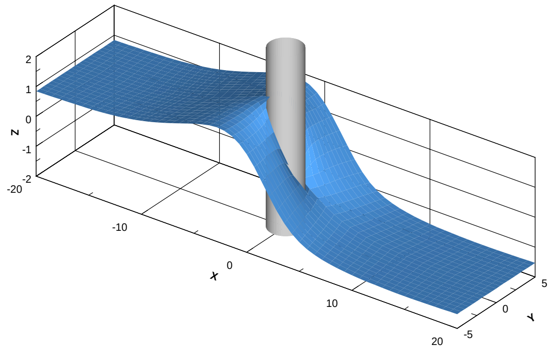


Fig 6. Wave profile for *4m* depth at *t=37.084 sec*

Diffraction by a vertical circular cylinder

Table1 Maximum run-up compared with numerical results of Ferrant in [11]

H/h	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
F.E.M.	1.90	2.12	2.38	2.68	2.99
Ferrant (1998)	1.87	2.06	2.38	N/A	N/A

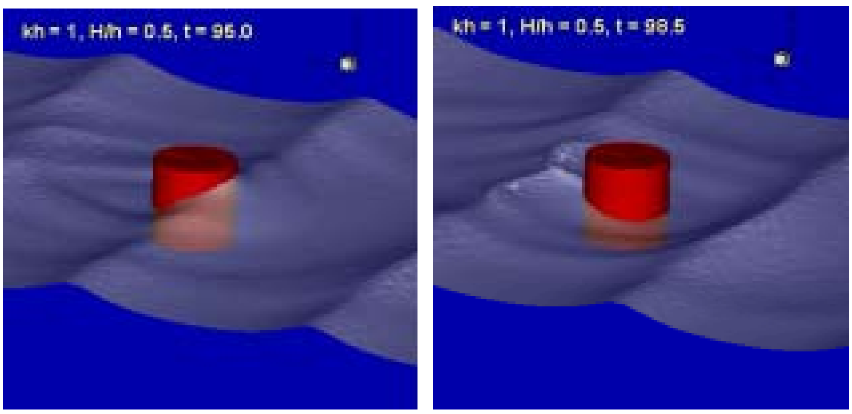


Figure 7. Snapshots of surface elevation around a cylinder.

CLOSING REMARKS

In paper the applications of the finite element method to various nonlinear free surface flow problems are described. The finite element method has been proven to be a useful method of solution for highly nonlinear free surface flow problems.

Acknowledgments

This work was supported by a Grant No.(R01-2000-000-00321-0) from Korea Science & Engineering Foundation, the Korea Research Foundation Grant.(KRF-2002-005-D00033),Research Institute of Marine Systems Engineering, college of engineering, Seoul National University. We thank Dr. Jo-Hyun Kyoung for his assistance during the preparation of this manuscript.

REFERENCES

1. Bai, K. J. & Kim, J. W., “FEM on Nonlinear Free Surface Flow,” Advances in Fluid Mechanics Series, Chapter 3, *Numerical Modeling in Fluid Stricture Interaction*, WIT Press 2005 (In print)
2. Bai, K. J., “A localized finite-element method for two-dimensional steady potential flows with a free surface,” *Journal of Ship Research*, Vol. 22, No. 4, 1978, pp. 216-230.
3. Bai, K. J. and Han, J.H., “A Localized Finite-Element Method for the Nonlinear Steady Waves due to a Two- Dimensional Hydrofoil,” *Journal of Ship Research*, Vol. 38, No. 1, March, 1994, pp. 42-51.
4. Milgram, J.H., “The Motion of a Fluid in a Cylindrical Container with a Free Surface Following Vertical Impact” *J. of F1uid Mech.* Vol. 37, Part 3, 1969, pp. 435-448.
5. Bai, K.J. Kim, J.W. , and Kai, Y.H., “Numerical Computations for a Nonlinear Free Surface Flow Problem,” *Proc. 5th Int. Conf. on Numerical Ship Hydrodynamics*, pp. 403-418, Hiroshima, Japan, 24-28 Sept 1989.
6. Bai, K.J., Kyoung, J. H., and Kim, J.W ., “A Numerical Computations for a Nonlinear Free-Surface Problem in Shallow Water,” *Journal of Offshore Mechanics and Arctic Engineering*, Vol. 125, No. 1, Feb, 2003, pp. 33-40.
7. Luke, J.C., “A variational principle for a fluid with a free surface.” *J. Fluid. Mech.* Vol. 27, 1967, pp. 395-397
8. Miles, J.W., “On Hamilton's Principle for Surface Waves” *J. Fluid Mech.* Vol. 83, 1977, pp. 153-158
9. Kyoung, Jo Hyun., Kim, Jang Whan, and Bai, Kwang June, “A Finite Element Method for the Nonlinear Sloshing Problem with a Bottom-Mounted Vertical Cylinder”, *OMAE*, Cancun, Mexico, June 8-13, 2003
10. Kim, J. W., Kyoung, J. H., Ertekin, R. C. and Bai, K. J., “A Finite-Element Computation of Wave-Structure Interaction by Steep Stokes Waves,” Submitted to *Journal of Waterway, Port, Coastal, and Ocean Engineering*, 2003.
11. Ferrant, P. “Runup on a Cylinder due to Waves and Current: Potential Flow Solution with Fully Nonlinear Boundary Conditions,” *Proc. 8th ISOPE Conf.*, Vol.3, pp.332-339, Montreal, Canada, 1998.