Discrete Maximum principle and Convergence Analysis for the Meshfree Point Collocation Method

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ABSTRACT

The discrete Laplacian operator is constructed using the meshfree point collocation method which will be called the strong meshfree Laplacian operator. To define the strong meshfree Laplacian operator, we use the generalized moving least square approximation[4,5,6], which can calculate the approximated derivatives of shape functions. Some types of the locally layered node distribution are defined in this paper and two specific domains are constructed on which we can distribute the locally layered nodes. On such types of nodes, the discrete maximum principle can be shown to hold through the representation formula for the strong meshfree Laplacian operator. The discrete maximum principle, together with the reproducing property of the meshfree approximations, results in discrete a priori estimate for the strong meshfree Laplacian operator on the nodal solution space. Furthermore, the a priori estimate we have obtained enables to prove the existence and the uniqueness of the numerical solution and plays a central role in achieving the convergence results for the Poisson problem with Dirichlet boundary condition on nodal solution space.
The order of convergence of the nodal solutions can be raised up to $O(h^2)$ on the proposed type of nodes in specific domains. For the generally shaped domains immersed in the previous domains, we can obtain the first order convergence result of $O(h)$. We know that finite difference methods and finite element methods[1,3] have some sort of discrete maximum principle for elliptic partial differential equation. For meshfree strong form collocation methods, no one has established any theoretical results until this paper. This is an important aspect, and this is the first paper that deals with the theoretical foundation of meshfree collocation method.
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