ENRICHED MESHFREE COLLOCATION METHOD FOR ELASTIC FRACTURE

Sang-Ho Lee [1], Young-Cheol Yoon [1] and Myoung-Won Kim[2]

1) *School of Civil and Environmental Engineering, Yonsei University, Seoul 120-749, KOREA*
2) *Department of Mathematics, Yonsei University, Seoul 120-749, KOREA*

Corresponding Author: Sang-Ho Lee, lee@yonsei.ac.kr

ABSTRACT

A meshfree collocation method with intrinsic enrichment for solving elastic crack problems is presented. A diffuse derivative approximation is applied in conjunction with an intrinsic enrichment of the near-tip asymptotic fields and a polynomial basis. These diffuse derivatives of the approximation do not require computing the derivative of the weight function or that of the moment matrix. The local behavior of the near-tip stresses is successfully captured so that the stress intensity factors can be accurately computed.

ENRICHED DIFFUSE DERIVATIVE APPROXIMATION

For a cracked body Ω, the displacement can be decomposed into the regular and singular parts by

$$u(\boldsymbol{x}) := u^R(\boldsymbol{x}) + u^S(\boldsymbol{x}) \tag{1}$$

The singular part $u^S(\boldsymbol{x})$ will be designed to exhibit the asymptotic behavior as follow

$$u_i(r,\theta) \sim r^{1/2} F_i(\theta), \qquad i = 1,\cdots,n \tag{2}$$

where $F_i(\theta)$ is determined by factors such as geometry, crack length and loading. On the other hand, the regular part of $u(\boldsymbol{x})$ is given by

$$u^R(x) := u(x) - u^S(x) \in C^m(\bar{\Omega}) \tag{3}$$

The regular part can be approximated by a polynomial basis; the singular part is expressed by adding a singular enrichment function [1,2] to the basis set. Then, the enriched local approximation defined by

$$u_L^{en}(\boldsymbol{x},\bar{\boldsymbol{x}}) := u_L^R(\boldsymbol{x},\bar{\boldsymbol{x}}) + u_L^S(\boldsymbol{x},\bar{\boldsymbol{x}}) \tag{4}$$

Now, let us consider a diffuse operator defined by

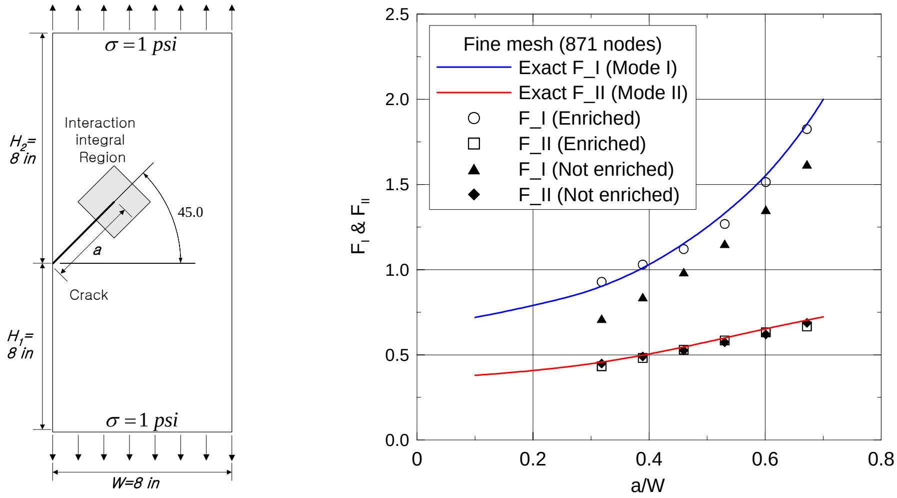
$$\mathfrak{D}_x^{\boldsymbol{\alpha}}(f(\boldsymbol{x},\bar{\boldsymbol{x}})) := \lim_{\bar{x}\to x}\left\{D_x^{\boldsymbol{\alpha}}(f(\boldsymbol{x},\bar{\boldsymbol{x}}))\right\} = \lim_{\bar{x}\to x}\left\{\frac{\partial^{|\boldsymbol{\alpha}|}\left(f(\boldsymbol{x},\bar{\boldsymbol{x}})\right)}{\partial x_1^{\alpha_1}\partial x_2^{\alpha_2}\cdots\partial x_n^{\alpha_n}}\right\} \tag{5}$$

The diffuse derivative is a good approximation of the derivative of $u(\boldsymbol{x})$ [3]. Thus, the derivatives of $u(\boldsymbol{x})$ can be approximated by the diffuse derivative:

$$D_x^{\boldsymbol{\alpha}} u(\boldsymbol{x}) \approx \mathfrak{D}_x^{\boldsymbol{\alpha}} u(\boldsymbol{x}) \tag{6}$$

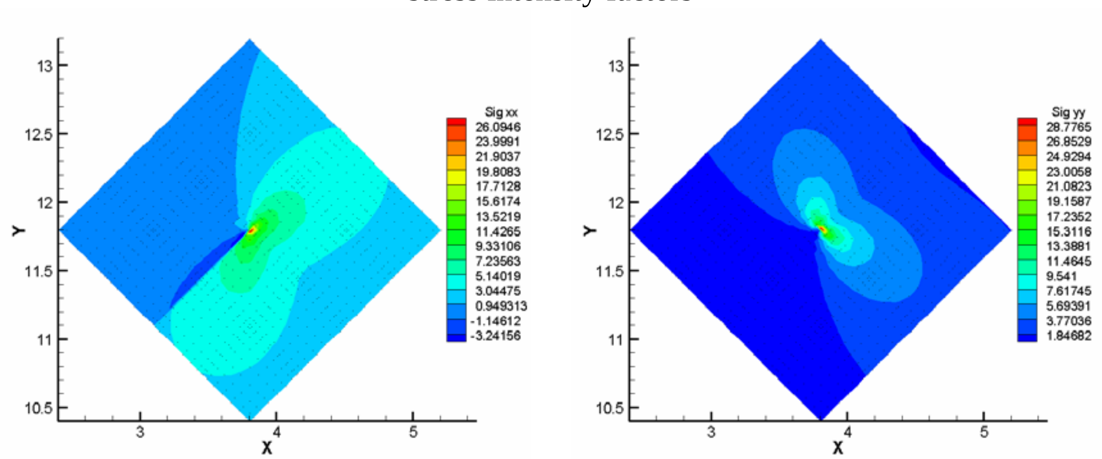
NUMERICAL EXAMPLE

A slant edge crack is considered to evaluate the methods for mixed mode cracks. Mode I and II stress intensity factors are evaluated for various lengths of an inclined crack. The plate is subjected to uni-axial tension of $\sigma = 1\,psi$. (See Figure 1(a)) In Figure 1(b), stress intensity factors for mode 1 and mode 2 are plotted as a function of crack length (a/W) as compared with the reference solution [4]. The enriched diffuse derivative approximation gives much better solutions than the unenriched approximation although the improvements for the mode II stress intensity factors are not significant. Similarly, the enriched approximation effectively captures the singular behavior of the stresses. (see Figure 2)



(a) (b)

Figure 1. (a) Problem description - edge slant cracked plate (b) normalized mode I and II stress intensity factors



(a) (b)

Figure 2. Contour plots for the stresses near the tip of slant crack (a) $\sigma_{11}(\boldsymbol{x})$ (b) $\sigma_{22}(\boldsymbol{x})$

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