A Simple Approach for Stochastic Interest Rate Option Pricing Model

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ABSTRACT

In this paper we present two kinds of the stochastic interest rate option pricing model. One is a bond numeraie approach and another is an approach of Merton type (1973). The Merton's approach is the most general one to incorporate the stochastic interest rate with option pricing model, but the implied volatility can not be calculated since volatilities and covariance of underlying asset and interest rate are mixed. However, the bond numeraie approach not only is simple but takes account into stochastic interest rate. It also makes calculation of implied volatility possible. One disadvantage can not be applicable for value zero asset like a futures contract.

1. Bond Numeraire Approcah

Under a continuous time economy with the complete and frictionless market, we evaluate a European call option with strike price K expiring at time T. Let $S(t)$ be the price of an underlying asset at time t. The underlying asset price normalized by the bond price, $P(t,T)$ is defined by $S(t)/B(t,T)$ where $B(t,T)$ is the price of the risk-free zero coupon bond with a payoff of \$1 at the maturity T which is the same date with the option's expiration date. Different from the Black-Scholes option pricing model (1973), we assume that the fractional change of the normalized underlying asset price follows one factor diffusion process, i.e.,

$$\frac{dP(t)}{P(t)} = \mu dt + \sigma dW(t)$$

where $dW(t)$ is a Wiener process, μ is the instantaneous expected rate and σ is the standard deviation of the fractional change of the normalized underlying asset price.

Then the normalized option by a bond price $V(P,\tau) = C(\tau, S, B(\tau))/B(\tau)$ satisfies

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V}{\partial P^2} = \frac{\partial V}{\partial \tau}.$$

At expiration, $S(T) = P(T,T)$ and so the payoff of the option will be

$$\max[S - K, 0] = \max[P - K, 0]$$

With an appropriate regularity condition the heat equation above can be solved and formula is obtained as follows :

$$C(t, S(t)) = S(t)N(d_1) - KB(t,T)N(d_2).$$

where $N(\bullet)$ is the cumulative normal distribution function and

$$d_1 = \frac{\ln(S/K) - \ln B(\tau) + 1/2\sigma^2 (T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t}.$$

As a corollary, Black-Scholes formula follows when the interest rate is constant.

2. Merton's Approach for value zero asset

In this case, interest rate process should be defined. If we assume that the returns of underlying asset and bond price dynamic follow lognormal diffusion process then the stochastic interest rate version of option pricing formula of Black model for futures option can be obtained.