

Optimal Control Theory Applied to a Difference Equation Model of Cardiopulmonary Resuscitation with Chest Compression Only

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ABSTRACT

Each year, more than 250,000 people die from cardiac arrest in the USA alone. Despite widespread use of cardiopulmonary resuscitation (CPR), the number of patients surviving from cardiac arrest remains low. Indeed, the rate of survival for CPR performed out of the hospital is 3%, while for patients who undergo cardiac arrest in the hospital, the rate of survival is 10-15%. One of the reasons for this situation is that the practical technique of CPR has changed little since the 1960's. The standard and various alternative CPR techniques such as interposed abdominal compression, active compression-decompression, and Lifestick CPR have been represented in various mathematical models [1, 2, 3]. Here, we illustrate our approach on a model of *standard* CPR, performed manually or by a mechanical device. The goal of this research is to reconsider the traditional CPR technique and to suggest novel strategies for improving it, by using the optimal control methodology.

The techniques of optimal control are applied to a validated blood circulation model of cardiopulmonary resuscitation (CPR) by Babbs [1]. In his model, heart and blood vessels are represented as a network of resistance and compliances. Pressures in the chest and in the vascular compliances are computed from difference equations. Blood flows are computed from Ohm's law, accounting for the action of one-way valves in the heart. The circulation model describes the adult human circulation (hemodynamics) and consists of seven difference equations, with time as the discrete underlying variable. As a control input, we choose the pattern of the pressure within the chest. More precisely, this control is actually the forcing pressure developed inside the chest as a result of external compression or decompression by the rescuer. The optimum waveform of this forcing pressure as a function of time was determined from control theory. The optimum waveform maximized systemic perfusion pressure (SPP) between the thoracic aorta and the superior vena cava over a period of 13.3 sec of continuous chest compression.

The new aspect in this application is that the control values from the two previous time steps are used to calculate the pressures (state variables) at the current time step. We prove the existence and uniqueness of the optimal control and provide a new CPR strategy, with increased blood flow. The characterization of the optimal control is given in terms of the solutions of the circulation model and of the corresponding adjoint system. The numerical results show a significant increase in the blood flow as compared with standard CPR. We

applied optimal control theory to the problem of CPR without any preconceived notions about what the results should be. The fact that optimal control theory found a solution incorporating two strategies already discovered in animal experiments and clinical trials, namely HIC-CPR with high impulse compressions, and ACD-CPR with active compression and decompression of the chest is quite interesting, since no bias toward these particular solutions was included in the problem definition or in the mathematics.

The results of this first-ever application of optimal control theory to CPR are interesting—perhaps less so in the exact numerical values of the solution than in the strategy they imply namely the combination of high impulse chest compression with intermittent decompression, as well as the idea that the ratio of compressions to decompressions need not necessarily be 1:1. The high impulse compressions augment forward flow and the active decompressions increase venous return to the pump. Such strategies can be realistically implemented by human rescuers or by non-fatiguing mechanical devices.

REFERENCES

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