Return-Mapping Algorithm for Cyclic Loading Analysis of Damaged Structures

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ABSTRACT

In this paper new return-mapping algorithm for the continuum large crack model is proposed for analysis of damaged structures subjected to cyclic loading. The algorithm consists of one elastic predictor and two return-mapping correctors. The numerical test results show that the present algorithm works appropriately under cyclic loading and should be used in large crack problems to prevent excessive tensile plastic strain causing unrealistic results.

PLASTIC-DAMAGE MODEL

The mechanism of microcrack opening and closing behavior can be modeled as elastic stiffness recovery during elastic unloading from a tensile state to a compressive state. Using a multiplicative parameter $0 \le s \le 1$ on the tensile degradation variable D_t, we have the degradation damage variable $D = 1 - (1 - D_c(\boldsymbol{\kappa}))(1 - sD_t(\boldsymbol{\kappa}))$, where D_c is the compressive degradation variable. The total stress $\boldsymbol{\sigma}$ is determined in the form of:

$$\begin{aligned} \boldsymbol{\sigma} &= (1 - D)\bar{\boldsymbol{\sigma}} \\ &= (1 - D_c(\boldsymbol{\kappa}))(1 - sD_t(\boldsymbol{\kappa}))\mathbf{E}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) \end{aligned} \tag{1}$$

where $\bar{\boldsymbol{\sigma}} = \mathbf{E}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p)$ is the effective stress, $\mathbf{E}_0$ is the initial elastic stiffness tensor, $\boldsymbol{\varepsilon}^p$ is the plastic strain, and $\boldsymbol{\kappa}$ is the damage variable vector [1].

After a large amount of microcracking, the crack opening and closing mechanism becomes similar to discrete cracking. In this study it is assumed that the microcracks are joined to construct a discrete large crack if $\kappa_t \ge \kappa_{cr}$, where κ_{cr} is an empirical value near unity. To model the large cracking, the evolution of the plastic strain caused by the tensile damage is stopped and the plastic strain increment is defined:

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} = (1 - s)\dot{\bar{\boldsymbol{\varepsilon}}}^{\mathrm{p}} \tag{2}$$

To make the effective stress based on Eq. 2 is admissible in the stress space it is necessary to introduce a new degradation variable D^{cr} and modify the effective stressing Eq. 1:

$$\tilde{\boldsymbol{\sigma}} = (1 - D^{cr})\mathbf{E}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{p}}) \tag{3}$$

The new degradation variable should be determined by the following Kuhn-Tucker type loading/unloading conditions such that:

$$\dot{D}^{cr} \ge 0; \quad \dot{D}^{cr} F(\tilde{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) = 0; \quad F(\tilde{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) \le 0 \tag{4}$$

where F is a yield function. Since during loading $F(\tilde{\boldsymbol{\sigma}},\boldsymbol{\kappa})=0$ which is a first-degree homogeneous function with respect to $\tilde{\boldsymbol{\sigma}}$, it is obtained:

$$D^{cr}=1-\frac{c_c(\boldsymbol{\kappa})}{f(\overline{\boldsymbol{\sigma}},\boldsymbol{\kappa})} \tag{5}$$

NUMERICAL ALGORITHM

To implement the large crack formulation described in the previous section numerically, a three-step return-mapping algorithm [2,3] based on the backward-Euler method is used in the present study. First, the following trial stress predictor is computed:

$$\tilde{\boldsymbol{\sigma}}_{n+1}^{tr}=(1-D_n^{cr})\mathbf{E}_0:(\boldsymbol{\varepsilon}_{n+1}-\boldsymbol{\varepsilon}_n^{\mathrm{p}}) \tag{6}$$

The trial stress is admissible as the effective stress at the current time step *n+1* if:

$$F(\tilde{\boldsymbol{\sigma}}_{n+1}^{tr},\boldsymbol{\kappa}_n)\equiv f(\tilde{\boldsymbol{\sigma}}_{n+1}^{tr},\boldsymbol{\kappa}_n)-c(\boldsymbol{\kappa}_n)<0 \tag{7}$$

where f is a yield surface; c is the cohesion parameter. Otherwise, the plastic and the crack damage correctors are required to make the effective stress admissible. At the plastic corrector step the plastic strain increment is discretized using the backward-Euler method:

$$\Delta\overline{\boldsymbol{\varepsilon}}^{\mathrm{p}}=\gamma_{n+1}\frac{\partial G_{n+1}}{\partial\overline{\boldsymbol{\sigma}}_{n+1}} \tag{8}$$

$$\boldsymbol{\varepsilon}_{n+1}^{\mathrm{p}}=\boldsymbol{\varepsilon}_n^{\mathrm{p}}+(1-s_{n+1})\Delta\overline{\boldsymbol{\varepsilon}}^{\mathrm{p}} \tag{9}$$

where the weight function s_{n+1} is computed if $\kappa_t\geq\kappa_{cr}$, and equal to be zero otherwise. At the next step the crack damage corrector makes the evaluated effective stress return back onto the yield surface:

$$D_{n+1}^{cr}=1-\frac{c_c(\boldsymbol{\kappa}_{n+1})}{f(\overline{\boldsymbol{\sigma}}_{n+1},\boldsymbol{\kappa}_{n+1})} \tag{10}$$

Accordingly, the modified effective stress in Eq. 1 becomes:

$$\tilde{\boldsymbol{\sigma}}=(1-D_{n+1}^{cr})\mathbf{E}_0:(\boldsymbol{\varepsilon}_{n+1}-\boldsymbol{\varepsilon}_{n+1}^{\mathrm{p}}) \tag{11}$$

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