EXACT DYNAMIC STIFFNESS MATRIX OF NON-SYMMETRIC THIN-WALLED CURVED BEAMS

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ABSTRACT

Firstly equations of motion and force-deformation relations are rigorously derived from the total potential energy for a shear deformable thin-walled non-symmetric curved beam element. Next a system of linear algebraic equations are constructed by introducing 14 displacement parameters and transforming the second order simultaneous differential equations into the first order simultaneous differential equations. And then explicit expressions for displacements are numerically evaluated via eigensolutions and the exact 14 × 14 dynamic element stiffness matrix is determined using force-deformation relations.

EQUATIONS OF MOTION

Allowing the shear deformation, the rotary inertia and the thickness-curvature effect, the elastic stain and kinetic energies [1] of thin-walled curved beams with non-symmetric cross section can be written as follows:

$$\begin{aligned}\Pi_E = \frac{1}{2}\int_0^L \Bigg[& EA\left(U_x' + \frac{U_z}{R}\right)^2 + E\hat{I}_2\left(\omega_2' - \frac{U_x'}{R} - \frac{U_z}{R^2}\right)^2 + E\hat{I}_3\left(\omega_3' - \frac{\omega_1}{R}\right)^2 + E\hat{I}_\phi f'^2 \\ & + 2E\widehat{I_{\phi 2}}\left(\omega_2' - \frac{U_x'}{R} - \frac{U_z}{R^2}\right) f' - 2E\widehat{I_{\phi 3}}\left(\omega_3' - \frac{\omega_1}{R}\right) f' - 2E\widehat{I_{23}}\left(\omega_3' - \frac{\omega_1}{R}\right)\left(\omega_2' - \frac{U_x'}{R} - \frac{U_z}{R^2}\right) \\ & + GJ\left(\omega_1' + \frac{\omega_3}{R}\right)^2 + GA_2(U_y' - \omega_3)^2 + GA_3\left(U_z' - \frac{U_x}{R} + \omega_2\right)^2 + GA_r\left(\omega_1' + \frac{\omega_3}{R} + f\right)^2 \\ & + 2GA_{23}(U_y' - \omega_3)\left(U_z' - \frac{U_x}{R} + \omega_2\right) + 2GA_{2r}(U_y' - \omega_3)\left(\omega_1' + \frac{\omega_3}{R} + f\right) + 2GA_{3r}\left(U_z' - \frac{U_x}{R} + \omega_2\right)\left(\omega_1' + \frac{\omega_3}{R} + f\right)\Bigg] dx_1\end{aligned} \tag{1a}$$

and

$$\begin{aligned}\Pi_M = \frac{1}{2}\rho\omega^2\int_0^L \Bigg[& A(U_x^2 + U_y^2 + U_z^2) + \tilde{I}_o\omega_1^2 + \tilde{I}_2\omega_2^2 + \tilde{I}_3\omega_3^2 + \tilde{I}_\phi f^2 + 2\frac{I_2}{R}(U_x\omega_2 - U_y\omega_1) \\ & - 2\frac{I_{23}}{R}(U_x\omega_3 - U_z\omega_1) - 2\widetilde{I_{23}}\omega_2\omega_3 + 2\widetilde{I_{\phi 2}}\omega_2 f - 2\widetilde{I_{\phi 3}}\omega_3 f + 2\frac{I_{\phi 2}}{R}U_x f\Bigg] dx_1\end{aligned} \tag{1b}$$

Now, equations of motion and force-deformation relations for curved beams may be derived by variation of Eq. (1) with respect to seven displacements.

EXACT DYNAMIC ELEMENT STIFFNESS MATRIX

In order to transform equations of motion into a set of the first order ordinary differential equations, a displacement state vector composed of 14 displacement parameters is defined by

$$d(x) = \langle d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}, d_{11}, d_{12}, d_{13}, d_{14} \rangle^T$$
$$= \langle U_x, U_x', U_y, U_y', \omega_3, \omega_3', U_z, U_z', \omega_2, \omega_2', \omega_1, \omega_1', f, f' \rangle^T \tag{2}$$

Using Eq. (2), equations of motion are transformed into the first order simultaneous ordinary differential equations with constant coefficients.

$$A\, d' = B\, d \tag{3}$$

Solving the eigenproblem with non-symmetric matrix in order to compute the homogeneous solution of the simultaneous differential equation (3), the general solution of Eq. (3) may be represented by the linear combination of eigenvectors with complex exponential functions as follows:

$$d(x) = \sum_{i=1}^{14} a_i\, Z_i\, e^{\lambda_i x} \tag{4}$$

Next it is necessary to represent complex coefficient vector $\boldsymbol{a}$ with respect to 14 nodal displacement components. And then elimination of a from Eq. (4) yields the exact displacement state vector. Finally the exact dynamic stiffness matrix K_d of a curved beam element with non-symmetric cross sections is determined by evaluating nodal forces at ends of element $(x = 0, l)$ using exact force-deformation relations.

NUMERICAL EXAMPLES

The clamped curved beam is analyzed. In Table 1, the spatially coupled natural frequencies by this study are compared with FE solutions with shear effect and the results from 300 nine-noded shell elements of ABAQUS.

Table 1. Spatially coupled natural frequencies of clamped curved beam

Mode	This study	FEM with shear deformation		ABAQUS
		4	20	
6	13.325	13.484	13.325	13.255
7	16.734	21.552	16.743	16.971
8	20.701	22.738	20.705	20.937
9	23.958	27.914	23.988	24.271
10	27.189	40.876	27.190	26.450

CONCLUSIONS

Through the numerical examples, it is found that the numerical results evaluated by **only one element** are in a good agreement with the FE solutions using the conventional thin-walled beam elements and ABAQUS's shell elements. Consequently it is judged that the present numerical procedure provides an efficient method for the numerical evaluation of exact dynamic element stiffness matrices of thin-walled curved beam-columns.

REFERENCES

1. Kim, N. I. and Kim, M. Y. “Spatial free vibration of shear deformable circular curved beams with non-symmetric thin-walled sections,” *Journal of Sound and Vibration*, Vol. 276, 2004, pp. 245-271.