Complexity analysis on $P_*(\kappa)$ LCP

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ABSTRACT

In this paper we propose a new primal-dual large-update path-following interior point algorithm for $P_*(\kappa)$ LCPs. We generalize the analysis of BER's primal-dual interior point algorithm for LP to $P_*(\kappa)$ LCPs. New search directions and proximity measures are proposed based on a kernel function. We show that if a strictly feasible starting point is available, then the new large-update primal-dual algorithms for solving $P_*(\kappa)$ LCPs have the similar polynomial complexity for LO which is the best complexity for $P_*(\kappa)$ LCPs.

INTRODUCTION

We consider the following linear complementarity problem (LCP) as follows :

$$\begin{cases} s = Mx + q, \\ xs = 0, \\ x \geq 0, \quad s \geq 0, \end{cases} \tag{LCP}$$

where $M \in R^{n\times n}$ is a $P_*(\kappa)$ matrix and $q \in R^n$.

Linear complementarity problems have many applications in mathematical programming and equilibrium problems. Indeed, it is known that by exploiting the first-order optimality conditions of the optimization problem, any differentiable convex program can be formulated into a monotone complementarity problem. And variational inequality problems are widely used in the study of equilibrium in, e.g., economics, transportation planning and game theory. And variational inequality problems have a close connection to the LCPs. The reader can refer [2] for the basic theory, algorithms and applications.

Miao ([4]) extended the MTY predictor-corrector method for $P_*(\kappa)$ LCPs. His algorithm uses the l_2 neighborhood of the central path and has $O((1+\kappa)\sqrt{n}L)$ iteration complexity. Recently, Illés and Nagy [3] give a version of the Mizuno-Todd-Ye predictor-corrector interior point algorithm for the $P_*(\kappa)$-matrix linear complementarity problem and show that the complexity of the algorithm is $O((1+\kappa)^{\frac{3}{2}}\sqrt{n}L)$. They choose τ and $\tau^{'}$ neighborhood parameters in such a way that a predictor step following by one corrector step at each iteration. For larger value of κ the values of τ and $\tau^{'}$ are fastly decreasing, therefore the constant in the complexity result is increasing.

Most of polynomial-time interior point algorithms for LO are based on the use of the logarithmic barrier function. Peng et al.([5]) introduced self-regular barrier functions for primal-dual interior-point methods for linear optimization and proved the best complexity for large-update primal-dual interior point methods for LO with some specific self regular barrier function. Recently Roos et al. [1] proposed a new primal-dual interior point method for LO based on a new proximity function which has a finite barrier. They give a new search direction depends on a univariate kernel function which is also used as proximity measure in the analysis of the algorithm and they get the best known iteration complexity.

We propose a new large-update primal-dual path following method which generalize BER's algorithm for LO to $P_*(\kappa)$ LCP and get the best known iteration complexity $O((1+2\kappa)\sqrt{n}log\frac{n}{\varepsilon})$ for $P_*(\kappa)$-matrix linear complementarity problem. At each iteration, the method select a target on the central path with a large-update from the the current iterate, and then new search direction is used based on a kernel function, followed by adaptively choosing the largest possible step size before leaving a neighborhood in which the proximity measure is given by the kernel function. Our analysis is easier than the mentioned previous results.

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