Eigenvalue Problem for Singular One-Dimensional p-Laplacian and Its Applications

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ABSTRACT

In this talk, we establish a sequence of eigenvalues of singular p-Laplacian boundary value problem:

$$\begin{cases} -\left(\varphi_p(u'(t))\right)' = \lambda h(t)\varphi_p(u(t)) \ \text{ a.e. } (0,1), \\ u(0) = u(1) = 0, \end{cases}$$

where $\varphi_p(x) = |x|^{p-2}x,\ p > 1,\ h \in L^1(0,1)$ and λ is a positive parameter. Next, we apply it to study global bifurcation phenomena of positive and sign-changing solutions for the following problem:

$$\begin{cases} -\left(\varphi_p(u'(t))\right)' = \lambda h(t) f\left(u(t)\right) \ \text{ a.e. } (0,1), \\ u(0) = u(1) = 0. \end{cases}$$

INTRODUCTION

Consider the following singular boundary value problem

$$\begin{cases} -\left(\varphi_p(u'(t))\right)' = \lambda h(t) f\left(u(t)\right) \ \text{ a.e. } (0,1), \\ u(0) = u(1) = 0, \end{cases} \tag{P^p_λ}$$

where $\varphi_p(x) = |x|^{p-2}x,\ p > 1$, and λ is a parameter. Here $h(t) \in L^1(0,1)$ is a nonnegative measurable function on $(0,1)$ that may be singular at $t = 0$ and/or $t = 1$, and the conditions on $f(u)$ will be given later.

By a solution of the problem (P^p_λ), we mean a function $u(t) \in C[0,1] \cap C^1(0,1)$ which satisfies both the equation on $(0,1)$ and the boundary condition in (P^p_λ). Moreover $\varphi_p(u'(t))$ is locally absolutely continuous in $(0,1)$ and the equality

$$-\left(\varphi_p(u'(t))\right)' = \lambda h(t) f\left(u(t)\right)$$

holds almost everywhere in $(0,1)$.

The problem (P^p_λ) was studied by several authors. Yang ([3]) showed the existence of the simple, isolated and positive eigenvalues which diverges to infinity for (P^p_λ) with restriction on h which is in $L^q(0,1), q > 1$, and $f(u) = \varphi_p(u), p \geq 2$. Notice that he excluded the case $1 < p < 2$. In [2], Sánchez obtained the existence of multiple positive solutions for the problem (P^p_λ) with a positive continuous function $h \in L^1(0,1)$, and super and sublinear cases of f. We denote the class

$$\mathcal{A} = \left\{ h \in C(0,1) : \int_0^{\frac{1}{2}} \varphi^{-1} \left(\int_s^{\frac{1}{2}} h(r)dr \right) ds + \int_{\frac{1}{2}}^1 \varphi^{-1} \left(\int_{\frac{1}{2}}^s h(r)dr \right) ds < \infty \right\}.$$

Clearly, we have $L^1(0,1) \subseteq \mathcal{A}$. Agarwal et al. established the existence of multiple positive solutions for the problem (P_λ^p) in [1] with $h \in \mathcal{A}$.

With help of the generalized Picone's type identity and Prüfer transformation, we shall establish the sequence $\{\lambda_k(p)\}$ of eigenvalues of the following problem:

$$\begin{cases} -\left(\varphi_p(u'(t))\right)' = \lambda h(t)\varphi_p(u(t)) \ \text{ a.e. } (0,1), \\ u(0) = u(1) = 0. \end{cases}$$

We shall show the existence of alternatives of global bifurcation at $(\lambda_k(p), 0)$ in the sense of Rabinowitz and the unboundedness of a subcontinuum. Finally, we shall sketch the shape of a subcontinuum of positive solutions from $(\lambda_1, 0)$ and of sign-changing solutions from $(\lambda_k, 0), k \geq 2$.

REFERENCES

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