Detection of Material Instabilities Using Acoustic Tensor

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ABSTRACT

Material property of a continuum solid is generally described by a constitutive relations with the material tangent tensor. Due to this fact, studies have been done by many researchers to relate some unstable behavior of a solid to a special condition of its material tangent. One of that kind of behaviors is the 45 degree shear band of a specimen under pure tension loading and plane strain boundary conditions. Many experiments have shown that the same kind of dislocation occurs in a specimen under pure tension loading with plane stress boundary condition, but in a different direction. The main purpose of this paper is to identify the possibility of the analytic detection in this case.

INTRODUCTION

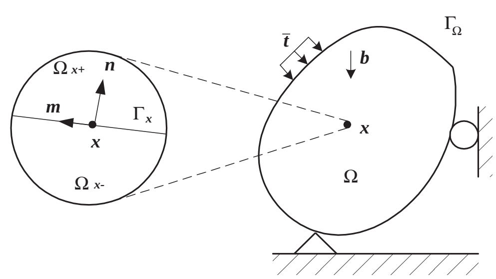

Figure 1. The local neighborhood

Unstable behavior of plastic solids often includes an abrupt change of deformation pattern from homogeneous one to highly localized intensive strain. Thus this behavior can be idealized as continuous displacement vector $\mathbf{u}$ and discontinuous strain field $\boldsymbol{\varepsilon}$ across a surface of discontinuity Γ_x (Fig.1). Let us define a jump operator $[[u_i]] := u_i^+ - u_i^-$. If we define a parameter s along the surface Γ_x, following two relations are always valid:

$$[[u_i]](s) = \frac{d}{ds}[[u_i]] = 0 \quad \forall s. \tag{1}$$

We introduce a unit vector $\mathbf{m}$ which is tangent to Γ_x. Then, the second part of the equation (1) leads to

$$\frac{d}{ds}[[\mathbf{u}]] = \quad \frac{d\mathbf{u}^+}{ds} - \frac{d\mathbf{u}^-}{ds} = \quad \nabla\mathbf{u}^+ \cdot \mathbf{m} - \nabla u_i^- \cdot \mathbf{m} = \quad [[\nabla\mathbf{u}]] \cdot \mathbf{m} = 0 \tag{2}$$

The last relation implies the gradient of displacement jump vector $[[\nabla \mathbf{u}]]$ to be as follows:

$$[[\nabla \mathbf{u}]] = g\mathbf{p} \otimes \mathbf{n}\,, \tag{3}$$

where $\mathbf{n}$ is a unit normal vector to $\mathbf{m}$, g is an arbitrary scalar and $\mathbf{p}$ is a unit vector.

For an arbitary small ball Ω_x including one point on Γ_x, we obtain the balance of linear momentum for three parts:

$$\int_{\partial\Omega} \boldsymbol{\sigma}\mathbf{n}\, d\Gamma = 0\,, \quad \int_{\partial\Omega+} \boldsymbol{\sigma}\mathbf{n}\, d\Gamma = 0\,, \quad \int_{\partial\Omega-} \boldsymbol{\sigma}\mathbf{n}\, d\Gamma = 0\,, \tag{4}$$

where the stress tensor $\boldsymbol{\sigma}$ is introduced. Due to the fact that $\Omega_x = \Omega_x^+ + \Omega_x^-$, it can be shown that

$$\int_{\partial\Omega} \boldsymbol{\sigma}\mathbf{n}\, d\Gamma = \int_{\partial\Omega+} \boldsymbol{\sigma}\mathbf{n}\, d\Gamma + \int_{\partial\Omega-} \boldsymbol{\sigma}\mathbf{n}\, d\Gamma\,, \tag{5}$$

which leads to a crucial relation

$$\int_{\Gamma_x} [[\boldsymbol{\sigma}]]\, \mathbf{n}\, d\Gamma = 0. \tag{6}$$

Because the choice of Ω_x is arbitrary, we obtain the final result

$$[[\boldsymbol{\sigma}]]\mathbf{n} = \mathbf{0}\,, \tag{7}$$

using the localization theorem.

The purpose of this paper is to identify possible instabilities of continuum solids using results presented above. Especially, more interest is put on pure tension case under plane stress boundary conditions.

REFERENCES

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