Computational Modeling of Waves in Composite Saturated Poroviscoelastic Media

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ABSTRACT

Wave propagation in composite porous materials has applications in many branches of science and technology, such as seismic methods in the presence of shaley sandstones [1], frozen or partially frozen sandstones [12,3,4], gas-hydrates in ocean-bottom sediments [5] and evaluation of the freezing conditions of foods by ultrasonic techniques [10].

A theory to describe wave propagation in frozen porous media was first presented by Leclaire *et al.* [8]. This model, valid for uniform porosity, predicts the existence of three compressional and two shear waves; the verification that additional (slow) waves can be observed in laboratory experiments was published by Leclaire *et al.* [9]. Later, Carcione and Tinivella [5] generalized this theory to include the interaction between the solid and ice particles and grain cementation with decreasing temperature. Also, Carcione *et al.* [1] applied this theory to study the acoustic properties of shaley sandstones, assuming that sand and clay are *non-welded* and form a continuous and inter-penetrating porous composite skeleton. Both frozen porous media and shaley sandstones are two examples of porous materials where the two solid phases are *weakly-coupled* or *non-welded*, i.e, both solids form a continuous and interacting composite structure, interchanging mechanical energy. Similar *weakly-coupled* formulations have previously been proposed. For instance, McCoy [11] has proposed a mixture theory appropriate for the combination of two *acoustic phases*.

This work presents a differential and numerical model to describe wave propagation in a heterogeneous poroviscolastic frame consisting of two weakly-coupled solid phases saturated by a single phase fluid. The equations of motion, stated in the space-frequency domain, generalizes that presented in [15] and [2] by the inclusion of solid matrix dissipation using a linear viscoelastic model and frequency dependent mass and viscous coupling coefficients. It also generalizes the models of Leclaire *et al.* [8] and Carcione *et al.* [5] for the case of uniform porosity, and consequently is the appropriate model to perform numerical simulation in heterogeneous materials.

The numerical procedures presented employ the nonconforming rectangular element defined in [7] to approximate the displacement vector in the solid phases. The dispersion analysis presented in [16] shows that employing this nonconforming element allows for a reduction in the number of points per wavelength necessary to reach a desired accuracy. On the other hand, the

displacement in the fluid phase is approximated by using the vector part of the Raviart-Thomas-Nedelec mixed finite element space of zero order, which is a conforming space [14,13].

The error analysis yields optimal *a priori* error estimates for the *global* standard and *hybridized* Galerkin methods.

Numerical simulation of waves in porous media is computationally expensive due to a large number of degrees of freedom needed to calculate wave fields accurately; the use of a domain decomposition iteration is a convenient approach to overcome this difficulty. Here we define a nonoverlapping domain decomposition iterative scheme and derive convergence results similar to those presented in [6] for solving second-order elliptic problems.

This iterative procedure was used for the simulation of waves in a sample of water saturated partially frozen Berea sandstone [2,5], perturbed by a point source at seismic frequencies. The sample has an interior plane interface defined by a change in ice content in the pores, and the snapshots of the generated wave fields show clearly the events associated with the different types of waves.

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