# A POSTERIORI ERROR ESTIMATORS FOR $P1$ NONCONFORMING APPROXIMATION OF QUASI-NEWTONIAN STOKES FLOWS

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**ABSTRACT**

In this work we derive some implicit error estimators for $P1$ nonconforming approximation of quasi-Newtonian Stokes flows obeying the Carreau law. These estimators are computed by solving local Neumann problems based on the recovered stress tensor. Numerical experiments are carried out to compare the performance of different error estimators.

## MODEL PROBLEM

The quasi-Newtonian Stokes flows are described by the equations

$$\begin{cases} -\operatorname{div}\mathcal{A}(\nabla \boldsymbol{u}) + \nabla p = \boldsymbol{f} & \text{in } \Omega, \\ \operatorname{div}\boldsymbol{u} = 0 & \text{in } \Omega, \\ \boldsymbol{u} = \mathbf{g} & \text{on } \Gamma, \end{cases} \tag{1}$$

where $\Omega$ is a bounded polygonal domain in $\mathbb{R}^2$ with boundary $\Gamma$, and $\boldsymbol{f} \in (L^2(\Omega))^2$ and $\mathbf{g} \in (H^{1/2}(\Gamma))^2$ are the given data. The Dirichlet boundary data $\mathbf{g}$ is required to satisfy the compatibility condition $\int_\Gamma \mathbf{g}\cdot\nu = 0$.

The tensor-valued function $\mathcal{A} : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$ is assumed to be Lipschitz continuous and strongly monotone, i.e., there exist positive constants $C_0, C_1$ such that, for all $\boldsymbol{\alpha}, \boldsymbol{\beta} \in \mathbb{R}^{2\times 2}$,

$$(\mathcal{A}(\boldsymbol{\alpha}) - \mathcal{A}(\boldsymbol{\beta})) : (\boldsymbol{\alpha} - \boldsymbol{\beta}) \geq C_0|\boldsymbol{\alpha} - \boldsymbol{\beta}|^2$$

and

$$|\mathcal{A}(\boldsymbol{\alpha}) - \mathcal{A}(\boldsymbol{\beta})| \leq C_1|\boldsymbol{\alpha} - \boldsymbol{\beta}|$$

with some constant $C_0, C_1 > 0$.

One particular case is the Carreau law with

$$\mathcal{A}(\boldsymbol{\alpha}) = \mu(|\boldsymbol{\alpha}|)\boldsymbol{\alpha} \qquad \forall \boldsymbol{\alpha} \in \mathbb{R}^{2\times 2},$$

where $\mu(t) = c_0 + c_1(1+t^2)^{(r-2)/2}$ with $c_0, c_1 > 0$ and $r \in [1, 2]$. The case $r = 2$ corresponds to the usual linear Stokes problem.

## MAIN RESULTS

Recently, Agouzal [1] derived the following simplified residual error estimator

$$\eta = \left( \sum_{T \in \mathcal{T}_h} h_T^2 \|\boldsymbol{f}\|_{0,T}^2 + \sum_{E \in \mathcal{E}_\Omega} h_E^{-1} \|[\![\boldsymbol{u}_h]\!]\|_{0,E}^2 + \sum_{E \in \mathcal{E}_\Gamma} h_E^{-1} \|\boldsymbol{u}_h - \mathbf{g}\|_{0,E}^2 \right)^{1/2}. \qquad (2)$$

for $P1$ nonconforming approximation of the quasi-Newtonian Stokes problem (1).

By rewriting the $P1$ nonconforming FEM in the form of the mixed finite volume method (see [2] for the second order elliptic equation), we compute a stress approximation $\boldsymbol{\sigma}_h$ in $\mathcal{RT}_0$ space by an explicit formula in terms of $(\boldsymbol{u}_h, p_h)$. Then we construct local Stokes and Poisson problems, where $\boldsymbol{\sigma}_h$ provides the necessary element and boundary data, which produce implicit error estimators.

The reliability and efficiency of the new estimators are established, and it is shown that, in some simple cases, the residual estimator (2) can be derived directly from the estimator based on Poisson problems. Numerical experiments are also carried out to compare the performance of different error estimators.

## REFERENCES


1. Agouzal, A., “A posteriori error estimator for finite element discretizations of quasi-Newtonian Stokes flows”, *Int. J. Numer. Anal. Model.*, Vol. 2, 2005, pp. 221–239.
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