Network disconnection problems in a centralized network

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ABSTRACT

Network disconnection problems are defined from a network attacker's point of view. In a centralized network where flows are supplied from a source node to the other nodes, called demand nodes, a network attacker aims to block the supply to demand nodes by destroying edges. As destroying edges incurs expenses, an attacker considers the following three different strategies. The first problem, denoted by (NDP1), is to maximize the unsatisfied flow demands while keeping the total cost of destroying the edges no more than a given budget. The second problem, denoted by (NDP2), is to minimize the edge destruction cost needed to make a certain amount of demand unsatisfied. The last problem, denoted by (NDP3), is to minimize the ratio of the edge destruction cost to the unsatisfied demand. In this paper, we derive the properties of the optimal solutions for the above three problems and show that the last one and the linear programming relaxations of the first two ones can be solved in a strongly polynomial time by using a parametric network flow algorithm.

To design a more survivable network, i.e., less vulnerable to network attacks, is one of the most important issues in constructing present-day communication networks. Cosares et al. [1] noted that survivable networks are generally more expensive than those with less robust designs, and thus it is essential to quantify the trade-offs between cost and survivability. Wu [11] and Grötschel, Monma, and Stoer [4] also identified network survivability as one of the most significant issues to be considered when designing communication networks. Therefore, network disconnection problems are useful to design a survivable network for a centralized network.

The first problem, (NDP1), has been addressed by Martel et al. [6] and has been shown that (NDP1) with unit costs and weights is NP-hard. Myung and Kim [8] present an integer programming formulation of (NDP1) and develop an algorithm that includes a preprocessing procedure and lower and upper bounding strategies. For (NDP2), no known research has been found but (NDP2) is closely related with (NDP1). For example, the decision versions of (NDP1) and (NDP2) are equivalent, which implies that (NDP2) is also NP-hard. The last problem, (NDP3), is a specific case of the sparsest cut problem. In the sparsest cut problem, each demand node is supplied from its own source node while all source nodes are identical in (NDP3). Although the sparsest cut problem is NP-hard, (NDP3) can be solved in polynomial time. However, to the best of our knowledge, no research has dealt with this specific case.

A couple of problems, not identical but similar to the network disconnection problems, have been introduced in the literature. Cunningham [2] and Gusfield [5] have considered a problem of minimizing the ratio of the edge destruction cost to the number of disconnected components. They have presented strongly polynomial time algorithms for this problem. Myung and Kim [8] have dealt with (NDP1) on a distributed network that has unit edge destruction cost. For the sparsest cut problem, many researchers have developed approximation algorithms. For surveys on these researches, see Shmoys [9] and Vazirani [10].

In this paper, we present our findings on the network disconnection problems. We investigate the properties of the optimal solutions and present integer programming formulations for the problems. Then we show that a parametric network flow algorithm solves in a strongly polynomial time the linear programming (LP) relaxations of the formulations for (NDP1) and (NDP2) and also show that the same algorithm finds an optimal solution of (NDP3) in the same time bound.

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