THE COMPRESSIBLE NAVIER–STOKES SYSTEM IN SINGULAR DOMAINS

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Our concern is with regularities of weak solutions of initial and boundary value problems for the evolution barotropic compressible Navier-Stokes system in a polygonal domain. Among many open problems relating to this, one of them is to extract the singular part of solution of the system near the corners or edges and to establish an increased regularity for the remainder. In the evolution case the space of singular functions is infinite dimensional. The coefficients of the singular functions are understood as functions of time variable. It is worth noting that solutions of the initial and boundary value problems for the compressible Navier-Stokes system have been mostly considered either in the whole space $\mathbf{R}^n$ or in the half spaces of $\mathbf{R}^n$ or in bounded domains with sufficiently smooth boundaries. It is wonder if the analysis can be applied in domains with singular boundaries. In many physical and engineering problems the compressible Navier-Stokes system is often considered in domains with corners and edges. In this paper we shall consider the system in polygonal domains.

We consider the following evolution compressible Navier-Stokes system in a bounded polygon $D \subset \mathbf{R}^2$ with Dirichlet boundary conditions

$$\begin{aligned} \rho \mathbf{u}_t - \mu\Delta\mathbf{u} - \nu\nabla\mathrm{div}\mathbf{u} + \rho(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = 0 &\quad \text{in } Q,\\ \rho_t + \mathrm{div}(\rho\mathbf{u}) = 0 &\quad \text{in } Q,\\ \mathbf{u} = 0 &\quad \text{on } \Sigma,\\ \mathbf{u}(\cdot,0) = \mathbf{u}_0,\ \rho(\cdot,0) = \rho_0 &\quad \text{on } D, \end{aligned}$$

where $Q := D \times (0,\,\mathrm{T})$ with a number $\mathrm{T} > 0$, $\Sigma := \partial D \times (0,\,\mathrm{T})$ is the lateral boundary of Q; $\mathbf{u}$ and p are the velocity and pressure variables; μ and ν are the coefficients of viscosity with $\mu > 0$ and $\mu + \nu > 0$; $\rho = \rho(p)$ is a strictly increasing smooth function of pressure p; $[\mathbf{u}_0, \rho_0]$ are given initial data with $\mathbf{u}_0\big|_{\partial D} = 0$ and $\rho_0 = \rho(p_0)$. We shall take the simpler case $\nu = 0$.

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