Stability of attractors formed by inertial particles in open chaotic flows

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ABSTRACT

Particles having finite mass and size advected in open chaotic flows can form attractors behind structures. Depending on the system parameters, the attractors can be chaotic or nonchaotic. But, how robust are these attractors? In particular, will small, random perturbations destroy the attractors? Here, we address this question by utilizing a prototype flow system: a cylinder in a two-dimensional incompressible flow, behind which the von Karman vortex street forms. We find that attractors formed by inertial particles behind the cylinder are fragile in that they can be destroyed by small, additive noise. However, the resulting chaotic transient can be superpersistent in the sense that its lifetime obeys an exponential-like scaling law with the noise amplitude, where the exponent in the exponential dependence can be large for small noise. This happens regardless of the nature of the original attractor, chaotic or nonchaotic. We present numerical evidence and a theory to explain this phenomenon. Our finding makes direct experimental observation of superpersistent chaotic transients feasible and it also has implications for problems of current concern such as the transport and trapping of chemically or biologically active particles in large-scale flows.

INTRODUCTION

The advective dynamics of idealized particles in two-dimensional, incompressible flows can be described as Hamiltonian [1,2]. In particular, consider such a flow characterized by a stream function $\Psi(x, y, t)$. For a particle with zero mass and size, its trajectory in the flow obeys the following equations:

$$\frac{dx}{dt} = \frac{\partial\Psi(x, y, t)}{\partial y}, \quad \frac{dy}{dt} = -\frac{\partial\Psi(x, y, t)}{\partial x}, \tag{1}$$

which are the standard Hamilton's equations of motion generated by the Hamiltonian $H(x, y, t) = \Psi(x, y, t)$. That is, the particle velocity $\mathbf{v}(x, y, t) = (dx/dt, dy/dt)$ follows exactly the flow velocity $\mathbf{u}(x, y, t)$, as given by the right-hand side of Eq. (1). This idealized picture changes completely when particles have finite mass and size. In this realistic case, the particle velocity is generally not the same as the flow velocity and the equations of motion are no longer Hamilton's equations. Maxley and Riley were the first to consider this problem [3] by deriving a set of equations for the particle velocity $\mathbf{v}(x, y, t)$, taking into account physical effects due to finite mass and size such as the buoyancy force, the Stokes drag, the added inertia effect, and other corrections [3–5]. The resulting dynamical system is no longer Hamiltonian but dissipative instead. As such, attractors can arise [5–9]. Considering that in an open Hamiltonian flow, ideal particles coming from the upper stream must necessarily go out of the region of interest in a finite amount of time, the formation of attractors of inertial particles is remarkable. Suppose these physical particles are biologically or chemically active. That they can be trapped permanently in some region in the physical space is of great interest or concern. A natural question is whether such attractors are structurally stable, i.e., whether they can persist under random perturbations.

In this paper, we are interested in the stability of attractors of inertial particles in open flows whose corresponding Lagrangian dynamics are chaotic. That is, for idealized particles, the equations of motion that they obey, namely Eq. (1), can exhibit chaos. We shall use the model of a two-dimensional flow passing a cylindrical obstacle, which was originally developed by Jung et al. [10]. Recent work by Benczik et al. [9] showed that for inertial particles in such a flow, attractors can be formed in the regions immediately behind the cylinder. We call such attractors *inertial attractors*. By varying a system parameter, periodic and chaotic attractors, and in fact a complete cascade of period-doubling bifurcations to chaos, can be observed.

Our result is that under small noise, inertial attractors are typically destroyed, leaving behind a transient. If the original attractor is chaotic, the transient is chaotic. However, we find that even if the attractor is nonchaotic, under noise the resulting transient can still be chaotic. In both cases, the average lifetime τ of the chaotic transient obeys the following scaling law with noise amplitude ε:

$$\tau \sim \exp\left(C\varepsilon^{-\gamma}\right), \quad \text{for } \varepsilon > \varepsilon_c, \tag{2}$$

where $C > 0$ and $\gamma > 0$ are constants, and ε_c is proportional to the minimum distance between the attractor and its basin boundary. The scaling law (2) is characteristic of *superpersistent chaotic transients*. By definition, a superpersistent chaotic transient is defined by the following scaling law for its lifetime [11]:

$$\tau \sim \exp\left(\alpha|p - p_c|^{-\gamma}\right), \tag{3}$$

where p is a system parameter, $\alpha > 0$ and $\gamma > 0$ are constants. We see that as p approaches the critical value p_c, the transient lifetime τ becomes superpersistent in the sense that the exponent in the exponential dependence diverges.

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