STABILIZATION FOR THE NONLINEAR DAMPED WAVE EQUATIONS IN EXTERIOR DOMAINS

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ABSTRACT

In this talk, we study the global existence of solutions and decay estimates of the energy for the nonlinear damped wave equation in exterior domains given by

$$(IBVP) \qquad \begin{cases} u_{tt} - \triangle u + \rho(x, u_t) = 0 & \text{in } \Omega \times (0, \infty) \\ u(x, 0) = u_0, \;\; u_t(x, 0) = u_1 & \text{in } \Omega \\ u(x, t) = 0 & \text{on } \partial\Omega \times (0, \infty) \end{cases}$$

Here we assume

$\Omega \subset \mathbb{R}^N$: an exterior domain such that $V \equiv \mathbb{R}^N \setminus \Omega$: compact

$\partial\Omega$: smooth, say C^3 class

$\rho(x, v)$: a function like $\rho(x, v) = a(x)g(v)$ with $a(x) \geq 0$ and $g'(v) \geq 0$, and differentiable $a.e.$ and nondecreasing function in $v(\neq 0)$ such that

$$(x, v) \in B_R^C \times \mathbb{R} \qquad \Longrightarrow \qquad \rho(x, v) = a(x)v$$

$$(x, v) \in \Omega_R \times \mathbb{R},\ |v| \leq 1 \qquad \Longrightarrow \qquad k_0 a(x)|v|^{p+2} \leq \rho(x, v)v \leq k_1 a(x)\{|v|^{p+2} + |v|^2\}$$

$$(x, v) \in \Omega_R \times \mathbb{R},\ |v| \geq 1 \qquad \Longrightarrow \qquad k_0 a(x)|v|^{q+2} \leq \rho(x, v)v \leq k_1 a(x)\{|v|^{q+2} + |v|^2\}$$

$$\Omega = \mathbb{R}^N \qquad \Longrightarrow \qquad \text{Cauchy problem.}$$

We remark that the dissipative term consists of two parts: The first part is nonlinear in suitable ball which contains the obstacle and is effective only in localized area; the second part is linear in the outside of the ball and is effective at infinity so we may call such dissipation the half-linear dissipation. We also note that the method of proof is based on the multiplier technique and on the unique continuation, and no geometrical condition is imposed on the boundary. Moreover an application is given.

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