LONG TIME ASYMPTOTICS AND A POTENTIAL COMPARISON TECHNIQUE.

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ABSTRACT

Recently a potential comparison technique has been developed for nonlinear diffusion equations and the optimal asymptotic convergence has been shown. This method is applicable to other problems after an appropriate adaptation. In the talk this method is introduced and some possible applications to convection, diffusion or p-laplacian are discussed.

A brief sketch of the potential comparison technique is as followings. Let $u, \tilde{u}$ be any two solutions and $U, \tilde{U}$ be their potentials. For the solutions of a convection equation $u_t + f(u)_x = 0$, for example, a primitive of the solution can be used as its potential. The first step is to show the potential comparison principle that is

$$U(x,t) \le \tilde{U}(x,t) \text{ for } t \ge t_0, x \in R \text{ if } U(x,t_0) \le \tilde{U}(x,t_0) \text{ for all } x \in R.$$

The second step is to obtain two constants $T, t_0 > 0$ such that

$$\tilde{U}(x, t_0 + T) \le U(x,t_0) \le \tilde{U}(x,0).$$

If these two steps are achieved, then we have

$$\|U(x,t) - \tilde{U}(x,t)\|_{L^\infty} \le \|\tilde{U}(x,t+T) - \tilde{U}(x,t-t_0)\|_{L^\infty}, \quad t \ge t_0.$$

The right hand side can be estimated explicitly. In fact we may show the convergence order of the right hand side formally using Taylor expansion. The last step is to transfer this estimate for potential different $U - \tilde{U}$ to the solution difference $u - \tilde{u}$. Finally we obtain optimal convergence of order $1/t$.