Optimization-based methods for multidisciplinary simulation and optimization

J. Lee [1] , M.D. Gunzburger [2] and Q. Du [3]

1) *Department of Mathematics, Yonsei University, Seoul 120-749, KOREA*

2) *Department of Mathematics, Florida State University, Tallehassee, FL 32306, USA*

3) *Department of Mathematics, Penn State University, University Park, PA 16802, USA*

ABSTRACT

We discuss algorithms for multidisciplinary simulation and optimization that efficiently couple existing single-discipline codes. The algorithms are based on a strategy in which unknown data at the interfaces is determined through an optimization process. The strategy allows for the user to select the data type at the interfaces for each discipline, so that the method can be tailored to existing codes. We focus on the fluid-structure interaction problem for which we describe the optimization-based methods.

INTRODUCTION

Multidisciplinary simulation and optimization problems arise in a variety of settings in which more than one media, or more than one mathematical model, or more than one dominant effect are present. The direct solution of such problems are a formidable challenge, especially whenever the individual disciplinary problems are themselves complex and whenever their solution are computationally intensive. For this reason, methods which, at the price of requiring an iterative procedure, uncouple the different disciplines are of interest. Here, we discuss uncoupling procedures which are based on using an optimization strategy.

A main virtue of our approach is that it allows for the user to use existing codes for each discipline as black boxes and only requires that the user write a simple code that effects the coupling between the disciplines. One reason we are able to do this is that our methodology allows for complete flexibility with regards to the boundary conditions imposed on each discipline. Another virtue of the optimization-based decoupling is that it allows for the use of efficient iterative strategies, e.g., the fast convergence of the iterative process by which solutions of a sequence of uncoupled problems converge to the solution of the coupled, multidisciplinary problem. Our methodology has other important virtues as well such as allowing for the use of mismatched grids and different discretization methods for each discipline.

Our ultimate goal is to develop robust, efficient, and accurate algorithms for the simulation of multidisciplinary problems, and then apply them to control and optimization problems as well. Here, for the sake of concreteness, we will describe the optimization-based domain decomposition method for fluid-structure interaction problems. The methodologies we describe for fluid-structure interaction problems can also be applied to many other multidisciplinary simulation and optimization problems.

Of course, the subjects of multidisciplinary simulation and optimization have been extensively studied in the past and continue to be the focus of much attention today. As a result, there is a vast literature on the subject. For example, one may consult the conference proceedings [1]-[6] and the references cited therein.

FLUID-STRUCTURE INTERACTION PROBLEMS

The interactions between fluid flows and solid structures immersed in the flow are of great interest. In practice, fluid-structure interactions are often modeled using elementary fluid models, e.g., involving loading functions, or ordinary differential equations, or linearized models such as panel methods, even when sophisticated models for the solid are used. There is considerable interest in including sophisticated fluid models as well. Although considerable effort has been devoted to such settings, the efficient and robust simulation and control of high-fidelity fluid-structure interactions is still not currently possible.

There are a number of different types of mathematical models for fluid-structure interactions that involve sophisticated fluids, e.g., the Navier-Stokes equations. Each model has a different regime of applicability. We briefly summarize three of the possibilities.

Rigid body motion of solids in a fluid flow. Here, the fluid sees the solids as moving rigid bodies. The motion of the solids is then simply described by a set of ordinary differential equations (6 for each body in three dimensions); the fluid motion is governed by unsteady flow equations, e.g., Navier-Stokes or Euler or potential. Coupling occurs since the region occupied by the fluid changes in time due to the motion of the solids and that motion is affected by the forces exerted by the fluid on the solids. Among the many applications of such fluid-structure interactions are store separation from aircraft, aircraft maneuvers including avoidance and pursuit strategies, response of aircraft to wind gusts and other sudden encounters, electromagnetic or acoustic nondestructive evaluation of pipelines and pipes by remote controlled vehicles swimming in the pipeline or pipe, and medical diagnosis and treatment using microdevices inserted into blood vessels, intestinal track, etc.

Elastic body motions in a fluid flow. Usually, the elastic motions of a solid body immersed in a fluid do not affect the fluid flow because the displacements in the solid are infinitesimal; the solid is affected by the fluid motion through the stress forces exerted by the the fluid on the solid. Such motions are uncoupled in the sense that one may solve for the fluid motion first, and then use that solution to define boundary data for the elastic motion of the solid. However, there is a very important situation for which the fluid and solid motions are fully coupled; this is when the solid is undergoing high frequency vibrations. In this case, although the elastic displacement of the solid may be small, the velocity is large. Then, the adherence (or the no penetration) condition that the fluid motion satisfies at the surface of the solid has the velocity of the solid as data. Among the many applications of such fluid-structure interactions are small amplitude flutter of wings, resonant vibrations of other aircraft components, and structural and other noise producing mechanisms.

Large displacement body motions in a fluid flow. When a solid body immersed in a fluid undergoes large displacement, then obviously the motions of the solid and fluid are coupled. Here, the difficulties, e.g., moving boundaries, associated with rigid body motions are present, but are made much more diffi- cult by the fact that the motion of the solid is now governed by a system of partial differential equations. Among the many applications of such fluidstructure interactions are flutter problems involving large amplitude vibrations and the buckling of solid structures under aerodynamic loads. Of course, the three types of interactions given above may be present in more complex situations. For example, rigid body motions and elastic motions may both be present when a wing is undergoing flutter. In this paper, we focus on the second case, i.e., elastic body motions in a fluid flow.

THE MODEL PROBLEM

We assume the fluid and solid occupy two adjacent, open, Lipschitz domains $\Omega_1 \subset \mathbb{R}^d$ and $\Omega_2 \subset \mathbb{R}^d$, respectively, where $d = 2$ or 3 is the space dimension. We denote by Ω the entire fluid-solid region under consideration, i.e., Ω is the interior of $\overline{\Omega}_1 \cup \overline{\Omega}_2$. Let $\Gamma_0 = \partial\Omega_1 \cap \partial\Omega_2$ denote the interface between the fluid and solid and let $\Gamma_1 = \partial\Omega_1 \setminus \Gamma_0$ and $\Gamma_2 = \partial\Omega_2 \setminus \Gamma_0$ respectively denote the parts of the fluid and solid boundaries excluding the interface Γ_0. For obvious reasons we assume that $\mathrm{meas}(\Gamma_1 \cup \Gamma_2) \neq 0$.

In the fluid region Ω_1, we apply the Stokes system

$$\begin{cases} \rho_1 \mathbf{v}_t + \nabla p - \mu_1 \nabla \cdot (\nabla \mathbf{v} + \nabla \mathbf{v}^T) = \rho_1 \mathbf{f}_1 & \text{in } \Omega_1 \\ \nabla \cdot \mathbf{v} = 0 & \text{in } \Omega_1 \\ \mathbf{v} = 0 & \text{on } \Gamma_1 \\ \mathbf{v}|_{t=0} = \mathbf{v}_0 & \text{in } \Omega_1, \end{cases} \tag{0.1}$$

where $\mathbf{v}$ denotes the fluid velocity, p the fluid pressure, $\mathbf{f}_1$ the given body force per unit mass, ρ_1 and μ_1 the constant fluid density and viscosity and $\mathbf{v}_0$ the given initial velocity.

In the solid region, we apply the equations of linear elasticity

$$\begin{cases} \rho_2 \mathbf{u}_{tt} - \mu_2 \nabla \cdot (\nabla \mathbf{u} + \nabla \mathbf{u}^T) - \lambda_2 \nabla(\nabla \cdot \mathbf{u}) = \rho_2 \mathbf{f}_2 & \text{in } \Omega_2 \\ \mathbf{u} = 0 & \text{on } \Gamma_2 \\ \mathbf{u}|_{t=0} = \mathbf{u}_0 \quad \text{and} \quad \mathbf{u}_t|_{t=0} = \mathbf{u}_1 & \text{in } \Omega_2, \end{cases} \tag{0.2}$$

where $\mathbf{u}$ denotes the displacement of the solid, $\mathbf{f}_2$ the given loading force per unit mass, μ_2 and λ_2 the Lamé constants, ρ_2 the constant solid density and $\mathbf{u}_0$ and $\mathbf{u}_1$ the given initial data.

Across the *fixed* interface Γ_0 between the fluid and solid, the velocity and stress vector are continuous. Thus, we have

$$\mathbf{u}_t = \mathbf{v} \qquad \text{on } \Gamma_0 \tag{0.3}$$

and

$$\mu_2(\nabla \mathbf{u} + \nabla \mathbf{u}^T) \cdot \mathbf{n}_2 + \lambda_2(\nabla \cdot \mathbf{u})\mathbf{n}_2 = p\mathbf{n}_1 - \mu_1(\nabla \mathbf{v} + \nabla \mathbf{v}^T) \cdot \mathbf{n}_1 \qquad \text{on } \Gamma_0, \tag{0.4}$$

where $\mathbf{n}_i$ is the outward-pointing unit normal vector along $\partial\Omega_i$, $i = 1, 2$.

We propose to solve the coupled fluid-structure interaction problem by solving a constrained optimization problem in which uncoupled fluid and solid systems act as constraints and in which the functional to be minimized measures the discrepancy in one of the interface conditions.

MULTIDISCIPLINARY OPTIMIZATION

We close with a few words about the use of the methods we have described in the setting of multidisciplinary optimization. In this a setting, we are given a functional that is to minimized (or maximized, depending on the application) and some parameters that can be varied in order to effect the optimization. We again want to solve this problem by a decomposition algorithm which uses already developed single discipline codes. Then, we are let to a multiobjective optimization problem involving the given functional and the functional which is artificially introduced in order to effect the decomposition into separate disciplines. The multiobjective minimization problem may be solved by a variety of means, the simplest of which is to form the single functional which is a weighted sum of two functionals. See [7] for an analytical and computational study of this approach in a simplified setting.

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