APPROXIMATE CONTROLLABILITY FOR SEMILINEAR RETARDED SYSTEMS DOMINATED BY THE LINEAR PART

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Abstract

Let H and V be two real separable Hilbert spaces such that V is a dense subspace of H. Identifying the antidual of H with H we may consider $V \subset H \subset V^*$.

In this raper we deal with the control problems for the semilinear parabolic type equation in H as follows.

$$\text{(RSE)} \qquad \begin{cases} \dfrac{d}{dt}x(t) = A_0 x(t) + \displaystyle\int_{-h}^{0} a(s) A_1 x(t+s) ds \\ \qquad\qquad + f(t, x(t)) + \Phi_0 u(t), \\ x(0) = g^0, \quad x(s) = g^1(s), \quad s \in [-h, 0). \end{cases}$$

Let A_0 be the operator associated with a sesquilinear form defined on $V \times V$ satisfying Gårding's inequality and A_1 be a bounded linear operator from V to V^* such that A_1 maps from $D(A_1)(\supset D(A_0)$ endowed with graphic norm of $A_0)$ to H continuously. Let $a(s)$ be a real valued Hölder continuous function on the interval $[-h, 0]$, where h is a fixed positive number. Let U be a complex Banach space and Φ_0 be a bounded linear operator from U to H.

With the aid of the solution semigroup, the equation (RSE) can be transposed onto an abstract evolution equation

$$\text{(TSE)} \qquad \frac{d}{dt} z(t) = Az(t) + F(t, z(t)) + \Phi u(t),$$

where $z(t) = (x(t), x_t(\cdot))$, $\Phi f = (\Phi_0 f, 0)$ and $F(t, z(t)) = (f(t, x(t)), 0)$. Here, the principal operator A is characterized by

$$Ag = (A_0 g^0 + \int_{-h}^{0} a(s) A_2 g^1(s) ds, \dot{g}^1), \quad g = (g^0, g^1) \in H \times L^2(-h, 0; V).$$

Thus, we are led studying the control problem in the space $Z \equiv H \times L^2(-h, 0; V)$. First, we give some preliminary results on the equation (RSE) with the Lipschitz continuity of nonlinear operator f from $\mathcal{R} \times V$ to H and establish the problem for the reguarality of solution of the equation (RSE).

We proceed to derive that any admissible control drives the state to zero asymptotically for linear system.

Under the discrete finite spectrum for the corresponding linear system we will show the equivalence relation between the approximately controllable for the system (RSE) and the stabilizability for solution of (RSE)(in the stability sense that for any $g \in Z$ there exists $u \in L^2(0, \infty; U)$ such that the solution $z(t)$ belongs to $L^2(0, \infty; Z)$).

Finally, from a condition for the range of the controller Φ_0 without the inequality condition as in [1,2], we establish to the approximate controllability for the semilinear system (RSE), and see that the necessary assumption is more flexible than one in [1,2,3].

Main results

Let $g \in Z$ and $x(t; g, f, u)$ be a solution of (RSE) associated with nonlinear term f and control u at time t. The segment x_t be given by $x_t(s; g, f, u) = x(t+s; g, f, u)$, $s \in [-h, 0)$. The solution semigroup $S(t)$ for the equation (RSE) is defined by

$$S(t)g = (x(t; g, 0, 0), x_t(\cdot; g, 0, 0))$$

where $x(t; g, 0, 0)$ is the solution of the equation (RSE) with $f(t, x) \equiv 0$ and $\Phi_0 \equiv 0$.

We assume

$$\sigma(A) \cap \{\lambda : \mathrm{Re}\, \lambda = 0\} = \emptyset$$

where $\sigma(A)$ is the spectrum of A.

Set

$$\sigma_+ = \sigma(A) \cap \{\lambda : \mathrm{Re}\, \lambda > 0\}, \quad \sigma_- = \sigma(A) \cap \{\lambda : \mathrm{Re}\, \lambda < 0\}.$$

We assume also that σ_+ is a finite and $\sup\{\mathrm{Re}\,\lambda : \lambda \in \sigma_-\} < 0$, that is,

$$\sigma_+ = \{\lambda_1, \dots, \lambda_N\}, \quad -\omega_0 = \sup\{\mathrm{Re}\,\lambda : \lambda \in \sigma_-\} < 0$$

and for each $j = 1, \dots, N$, the spectral projection

$$P_{\lambda_j} = \frac{1}{2\pi i} \int_{\Gamma_{\lambda_j}} (\lambda - A)^{-1} d\lambda$$

is an operator of finite rank, where Γ_{λ_j} is a small circle centered at λ_j such that it surrounds no point of $\sigma(A)$ except λ_j. As is well known λ_j is an eigenvalue of A and the generalized eigenspace corresponding to λ_j is given by

$$Z_{\lambda_j} = P_{\lambda_j} Z = \{P_{\lambda_j} u : u \in Z\}.$$

First, we deal with the equivalent conditions of the approximate controllability for the linear system in case where $f \equiv 0$ in the system (TSE):

$$\text{(TE)} \qquad z^{'}(t) = Az(t) + \Phi u(t), \quad z(0) = g.$$

We define reachable sets for the systems (TE) and (TSE) as follows.

$$L(T; g) = \{z(T; g, 0, u) : \; u \in L^2(0, T; U)\},$$
$$R(T; g) = \{z(T; g, f, u) : \; u \in L^2(0, T; U)\}.$$

Moreover, we introduce the unobservable subspace for the dual system of the system (TAE):

$$N(T) = \bigcap_{0 \le t \le T} \Phi^* S_{\mathrm{T}}(t).$$

Definition. *Let $\lambda \in \sigma_+$.*
(1) The system (TSE) is said to be $Z_\lambda-$approximately controllable (resp. approximately controllable) on $[0, T]$ if $\overline{R(T; g)} \subset Z_\lambda$ (resp. $\overline{R(T; g)} = Z$).
(2) The system (TAE) is $Z_{\overline{\lambda}}-$observable on $[0, T]$ (resp. observable) if $N(T) \cap Z^{\mathrm{T}}_{\overline{\lambda}} = \{0\}$ (resp. $N(T) \cap Z = \{0\}$).

Theorem 1. *The following statements are equivalent.*
(a) For any $g \in Z$ there exists an $u \in L^2(0,\infty;U)$ such that the mild solution z of (TSE) satisfies $z \in L^2(0,\infty;Z)$.
(b) The system of (TSE) is $Z_{\lambda_j}-$approximately controllable for every $\lambda_j \in \sigma_+$.
(c) The adjoint system of (TSE) is $Z_{\overline{\lambda_j}}-$observable for every $\lambda_j \in \sigma_+$.

We define the linear operator $\hat{S}$ from $L^2(0,T;Z)$ to Z by

$$\hat{S}p = \int_0^T S(T-s)p(s)ds$$

for $p \in L^2(0,T;Z)$. The system (TSE) is approximately controllable on $[0,T]$ if for any $\varepsilon > 0$ and $\xi_T \in Z$ there exists a control $u \in L^2(0,T;U)$ such that

$$||\xi_T - S(T)g - \hat{S}F(\cdot, z(\cdot;g)) - \hat{S}\Phi u|| < \varepsilon.$$

We need the following hypothesis:

For any $\varepsilon > 0$ and $p \in L^2(0,T;Z)$ there exists a $u \in L^2(0,T;U)$ such that

$$\text{(B)} \qquad \begin{cases} ||\hat{S}p - \hat{S}\Phi u|| < \varepsilon, \\ ||\Phi u||_{L^2(0,t;Z)} \le q_1 ||p||_{L^2(0,t;Z)}, \quad 0 \le t \le T. \end{cases}$$

where q_1 is a constant independent of p.

Theorem 2. *Under the assumption (B), the system (TSE) is approximately controllable on $[0,T]$.*

References

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