Generalized Second Order Symmetric Duality in Nondifferentiable Multiobjective Programming

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ABSTRACT

In this talk, motivated by Mond and Schechter [2], Hou and Yang [1], Suneja et al. [3] and Yang et al. [4], we formulate a pair of multiobjective generalized second order symmetric dual programs where the objective function contains a support function. Weak duality, strong duality and converse duality theorems are established under second order F-convexity and F-concavity assumptions. As special cases of our duality results, we give Mond-Weir type, Wolfe type duality theorems and some of the known results.

GENERALIZED SECOND ORDER SYMMETRIC DUALITY

Our models are as follows :

$(GSMP)$ Minimize $K(x,y,\lambda,w,p)$

$$= f(x,y) + (s(x|B))e - (y_J^T w)e - (y_I^T \nabla_{y_I}(\lambda^T f)(x,y))e$$
$$-(y_I^T \nabla_{yy_I}(\lambda^T f)(x,y)p)e - \tfrac{1}{2}(p^T \nabla_{yy}(\lambda^T f)(x,y)p)e$$

subject to

$$\nabla_y(\lambda^T f)(x,y) - w + \nabla_{yy}(\lambda^T f)(x,y)p \leqq 0 \ ,$$
$$y_J^T \nabla_{y_J}(\lambda^T f)(x,y) - y_J^T w + y_J^T \nabla_{yy_J}(\lambda^T f)(x,y)p \geqq 0 \ ,$$
$$w \in C, \quad \lambda > 0, \quad \lambda^T e = 1 \ ,$$

$(GSMD)$ Maximize $G(u,v,\lambda,z,r)$

$$= f(u,v) - (s(v|C))e + (u_B^T z)e - (u_A^T \nabla_{x_A}(\lambda^T f)(u,v))e$$
$$-(u_A^T \nabla_{xx_A}(\lambda^T f)(u,v)r)e - \tfrac{1}{2}(r^T \nabla_{xx}(\lambda^T f)(u,v)r)e$$

subject to

$$\nabla_x(\lambda^T f)(u,v) + z + \nabla_{xx}(\lambda^T f)(u,v)r \geqq 0 \ ,$$
$$u_B^T \nabla_{x_B}(\lambda^T f)(u,v) + u_B^T z + u_B^T \nabla_{xx_B}(\lambda^T f)(u,v)r \leqq 0 \ ,$$
$$z \in B, \quad \lambda > 0, \quad \lambda^T e = 1 \ ,$$

where

(i) f is a trice differentiable function from $\mathbb{R}^n \times \mathbb{R}^m$ to $\mathbb{R}^k$;
(ii) r, z are vectors in $\mathbb{R}^n$, p, w are vectors in $\mathbb{R}^m$;
(iii)λ and $e = (1, \cdots, 1)^T$ are vectors in $\mathbb{R}^k$;

(iv)B and C are compact convex sets in $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively ; and

(v)$N = \{1, 2, \cdots, n\}$, $M = \{1, 2, \cdots, m\}$, $A \subset N$, $I \subset M$,$N \setminus A = B$ and $M \setminus I = J$. Note that A, B, I, or J can be empty.

Weak Duality Theorem. Let (x, y, λ, w, p) be feasible for $(GSMP)$ and (u, v, λ, z, r) be feasible for $(GSMD)$. Assume that

(i)$f(., v) + ((.)^T z)e$ is second order F-convex in the 1st variable,

(ii)$f(x, .) - ((.)^T w)e$ is second order F-concave in the 2nd variable,

(iii)$F_{x,u}(a) + a^T u \geqq 0$ for all $a \in \mathbb{R}^n_+$, and

(iv)$G_{v,y}(b) + b^T y \geqq 0$ for all $b \in \mathbb{R}^m_+$.

Then $K(x, y, \lambda, w, p) \nleq G(u, v, \lambda, z, r)$.

Strong Duality Theorem. Let f be a thrice differentiable function from $\mathbb{R}^n \times \mathbb{R}^m$ to $\mathbb{R}^k$. Let $(\overline{x}, \overline{y}, \overline{\lambda}, \overline{w}, \overline{p})$ be an efficient solution for $(GSMP)$; fix $\lambda = \overline{\lambda}$ in $(GSMD)$ and suppose that

(i) $\nabla_{yy}(\overline{\lambda}^T f)(\overline{x}, \overline{y})$ is non-singular,

(ii) $\nabla_{y_J}(\overline{\lambda}^T f)(\overline{x}, \overline{y}) - \overline{w} + \nabla_{yy_J}(\overline{\lambda}^T f)(\overline{x}, \overline{y})\overline{p} \neq 0$,

(iii)the set $\{\nabla_{y_J} f_1(\overline{x}, \overline{y}), \nabla_{y_J} f_2(\overline{x}, \overline{y}), \cdots, \nabla_{y_J} f_k(\overline{x}, \overline{y}), \overline{w}\}$ is linearly independent, and

(iv)the matrix $\frac{\partial}{\partial y_i}(\nabla_{yy}(\overline{\lambda}^T f))$ is positive or negative definite,

for some $i \in I$, where $f = f(\overline{x}, \overline{y})$.

Then there exists $\overline{z} \in B$ such that $(\overline{x}, \overline{y}, \overline{\lambda}, \overline{z}, \overline{r} = 0)$ is a feasible solution for $(GSMD)$ and $K(\overline{x}, \overline{y}, \overline{\lambda}, \overline{w}, \overline{p}) = G(\overline{x}, \overline{y}, \overline{\lambda}, \overline{z}, \overline{r})$.

Moreover, if the hypotheses of Weak Duality Theorem are satisfied for all feasible solutions of $(GSMP)$ and $(GSMD)$, then $(\overline{x}, \overline{y}, \overline{\lambda}, \overline{z}, \overline{r})$ is a properly efficient solution for $(GSMD)$.

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