An Example for Vector Matrix Game

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ABSTRACT

A vector matrix game is defined by B_i's of real $n \times n$ skew smmetric matrices together with the Cartesian product $S_n \times S_n$ of the set of all the n-dimensional probability vectors S_n, that is, $S_n := \{x = (x_1, \cdots, x_n)^T \in \mathbb{R}^n : x_i \geqq 0, \sum_{i=1}^n x_i = 1\}$, where the symbol ^{T} denotes the transpose.

Vector matrix game can be regarded as a special case of vector sadle point problem which has been studied by many authors, and as a vector version of well-known (scalar) matrix game. Recently, we formulated a Wolfe type dual problem for a linear vector optimization problem and established equivalent relations between the dual problem and certain vector matrix game.

However, as far as we know, there are very few papers which directly treat with solutions for vector matrix game.

Now we define the following solution conceptions for vector matrix game:

We give the following conventions for vectors in the Euclidean space $\mathbb{R}^n$ for vectors $x := (x_1, \cdots, x_n)^T$ and $y := (y_1, \cdots, y_n)^T$;

$$x < y \text{ if and only if } x_i < y_i,\ i = 1, \cdots, n;$$
$$x \leq y \text{ if and only if } x \leqq y, \text{ and } x \neq y;$$
$$x \nless y \text{ is the negation of } x < y; \text{ and}$$
$$x \nleq y \text{ is the negation of } x \leq y.$$

We let $\overset{o}{S}_n = riS_n$, where riS_n is the relative interior of S_n.

Let $B_i,\ i = 1, \cdots, p$, be real $n \times n$ skew-symmetric matrices.

(1) A point $\bar{x} \in S_n$ is said to be a vector solution of vector matrix game $B_i,\ i = 1, \cdots, p$ if

$$(x^T B_1 \bar{x}, \cdots, x^T B_p \bar{x}) \nleq (\bar{x}^T B_1 \bar{x}, \cdots, \bar{x}^T B_p \bar{x}) \nleq (\bar{x}^T B_1 x, \cdots, \bar{x}^T B_p x)$$

for any $x \in S_n$.

(2) A point $\bar{x} \in S_n$ is said to be a weakly vector solution of vector matrix game $B_i,\ i = 1, \cdots, p$ if

$$(x^T B_1 \bar{x}, \cdots, x^T B_p \bar{x}) \nless (\bar{x}^T B_1 \bar{x}, \cdots, \bar{x}^T B_p \bar{x}) \nless (\bar{x}^T B_1 x, \cdots, \bar{x}^T B_p x)$$

for any $x \in S_n$.

(3) A point $(\bar{x}, \bar{y}) \in S_n \times S_n$ is said to be an efficient solution of vector matrix game $B_i,\ i = 1, \cdots, p$ if

$$(x^T B_1 \bar{y}, \cdots, x^T B_p \bar{y}) \ngeq (\bar{x}^T B_1 \bar{y}, \cdots, \bar{x}^T B_p \bar{y}) \ngeq (\bar{x}^T B_1 y, \cdots, \bar{x}^T B_p y)$$

for any $x,\ y \in S_n$.

(4) A point $(\bar{x}, \bar{y}) \in S_n \times S_n$ is said to be a weakly efficient solution of vector matrix game $B_i,\ i = 1, \cdots, p$ if

$$(x^T B_1 \bar{y}, \cdots, x^T B_p \bar{y}) \not> (\bar{x}^T B_1 \bar{y}, \cdots, \bar{x}^T B_p \bar{y}) \not> (\bar{x}^T B_1 y, \cdots, \bar{x}^T B_p y)$$

for any $x,\ y \in S_n$.

(5) A point $(\bar{x}, \bar{y}) \in S_n \times S_n$ is said to be a scalarizing solution of vector matrix game $B_i,\ i = 1, \cdots, p$ if there exists $\lambda \in \overset{o}{S}_p$ such that

$$x^T (\sum_{i=1}^{p} \lambda_i B_i) \bar{y} \leqq \bar{x}^T (\sum_{i=1}^{p} \lambda_i B_i) \bar{y} \leqq \bar{x}^T (\sum_{i=1}^{p} \lambda_i B_i) y$$

for any $x,\ y \in S_n$.

(6) A point $(\bar{x}, \bar{y}) \in S_n \times S_n$ is said to be a weakly scalarizing solution of vector matrix game $B_i,\ i = 1, \cdots, p$ if there exists $\lambda \in S_p$ such that

$$x^T (\sum_{i=1}^{p} \lambda_i B_i) \bar{y} \leqq \bar{x}^T (\sum_{i=1}^{p} \lambda_i B_i) \bar{y} \leqq \bar{x}^T (\sum_{i=1}^{p} \lambda_i B_i) y$$

for any $x,\ y \in S_n$.

We denote the set of all the vector solutions, the set of all the weakly vector solutions, the set of all the effcient solutions, the set of all the weakly efficient solutions, the set of all the scalarizing solutions and the set of all the weakly scalarizing solutios, for vector matrix game, by sol(VMG), sol(WVMG), sol(EVMG), sol(WEVMG), sol(SVMG) and sol(WSVMG), respectively.

Then it is clear from definitions that the following hold:

$$sol(\text{SVMG}) \subset sol(\text{EVMG}) \subset sol(\text{WEVMG})$$

and

$$sol(\text{WSVMG}) \subset sol(\text{WEVMG}).$$

However, It is not clear whether four kinds of solutions for a vector matrix game, which are described in (3)$-$(6) may be different or not. In this talk, using vector optimization techniques, we characterize solution sets for a vector matrix game, which are described in (1)$-$(6), and then we give an example showing that four kinds of solutions, which are described in (3)$-$(6), may be different.