On ϵ-Approximate Solutions for Convex Semidefinite Optimization Problems

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ABSTRACT

Many authors have studied ϵ-aaproximate solutions for several kinds of convex optimization problem ([1-3]). It is well known that constrainted qualifications should be imposed on convex optimization problems to get ϵ-optimality theorems and ϵ-saddle point theorems for ϵ-approximate solutions.

In this talk, we consider the convex semidefinite programming model problem:

$$\text{(SDP)} \qquad \text{Minimize} \quad f(x)$$
$$\text{subject to} \quad F_0 + \sum_{i=1}^{m} x_i F_i \succeq 0,$$

where $f : \mathbb{R}^m \to \mathbb{R}$ is a convex function, and for $i = 0, 1, \cdots, m$, $F_i \in S_n$, the space of $(n \times n)$ real symmetric matrics. The space S_n is partially ordered by the $L\ddot{o}wner$ order; that is, for $M, N \in S_n$, $M \succeq N$ if and only if $M - N$ is positive semidefinite. The inner product in S_n is defined by $(M, N) = Tr[MN]$, where $Tr[\cdot]$ is the trace operation. Let $S := \{M \in S_n \mid M \succeq 0\}$. Then

$$S^+ = \{\theta \in S_n \mid (\theta, Z) \geqq 0 \; \forall Z \in S\} = S.$$

Let $F(x) := F_0 + \sum_{i=1}^{m} x_i F_i$, $\hat{F}(x) = \sum_{i=1}^{m} x_i F_i$, $x = (x_1 \cdots, x_m) \in \mathbb{R}^m$. Then $\hat{F}$ is a linear operator from $\mathbb{R}^m \; to \, S_n$ and its dual is defined by

$$\hat{F}^*(Z) = (Tr[F_1 Z], \cdots, Tr[F_m Z])$$

for any $Z \in S_n$. Clearly, $A := \{x \in \mathbb{R}^m \mid F(x) \in S\}$ is the feasible set of (SDP).

Let $\epsilon \geqq 0$. Then $\bar{x} \in S$ is called an ϵ-approximate solution of (SDP) if for any $x \in A$,

$$f(x) + \epsilon \geqq f(\bar{x}).$$

When the objective function is linear and the corresponding matrices are diagonal, (SDP) becomes a linear optimization problem. The problem (SDP) includes many important applications in systems and control theory, approximate theory and combinatorial optimization ([4]).

Also, many kinds of optimization problems can be reduced to this problem (SDP). So semidefinite optimization problems have been intensively investigated ([5, 6]).

We formulate a Wolfe type dual problem for (SDP) as follows:

$$
\begin{array}{lll}
\text{(SDD)} & \text{Maximize} & f(x) - Tr[ZF(x)] \\
& \text{subject to} & 0 \in \partial_{\epsilon_0} f(x) - \hat{F}^*(Z) \\
& & Z \succeq 0 \\
& & \epsilon_0 \leqq \epsilon,
\end{array}
$$

where $\partial_{\epsilon_0} f(x)$ is the ϵ_0-subgradient of f at x.

In this talk, using techniques in [7, 8], we give ϵ-optimality theorems and ϵ-saddle point theorems for ϵ-approximate solutions of (SDP) which hold under weakened constraint qualification or which hold without any constraint qualification. Moreover, we formulate a Wolfe type dual problem for (SDP), and prove weak duality and strong duality between the primal problem and the dual problem, which hold under a weakened constraint qualification.

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