

Three-dimensional Flow Physics Analyses Using Multi-dimensional Limiting Process

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ABSTRACT

In this paper, we apply three-dimensional limiting process for three-dimensional flow physics analyses. The basic idea of multi-dimensional limiting condition is that the multi-dimensionally interpolated values at a vertex point should be within the maximum and minimum cell-average values of neighboring cells for the monotonic distribution. By applying the MLP (Multi-dimensional Limiting Process) to the three dimensional Euler and Navier-Stokes equations, we can achieve monotonic characteristics, which results in the enhancement of solution accuracy, convergence behavior.

INTRODUCTION

Accurate monotonic schemes for hyperbolic conservation laws are developed based on one-dimensional flow physics through the analysis of TVD limiters [1], [2]. It shows the complete monotonic and accurate distribution in a one-dimensional discontinuity. However, if they are applied to a multi-dimensional problem, the interpolated property, without considering the effect of other flow directions, certainly leads to a non-monotonic distribution. In order to find out the monotonicity condition for multi-dimension, Kim et al. [3] extended the one-dimensional monotonic condition to two-dimensional problem and presented the two-dimensional limiting condition successfully. With the limiting condition, a multi-dimensional limiting process (MLP) is proposed which gives more accurate results for two-dimensional Euler and Navier-Stokes equations. It was the approach which prompted the work of the present paper. Basically, it extends the idea of MLP to three-dimensional problem. Thus, in this paper, we introduce a three-dimensional limiting condition and present the numerical investigation of test cases which include complex physical phenomena.

MLP FOR THREE-DIMENSIONAL FLOWS

After the three-dimensional limiting condition is applied, the general form of MLP is written as follows

$$\Phi_{i+\frac{1}{2},L} = \Phi_i + 0.5 \max(0, \min(\alpha_L, \alpha_{L,R}, \beta_L)) \Delta \Phi_{i-\frac{1}{2}} \quad (1)$$

$$\Phi_{i+\frac{1}{2},R} = \Phi_{i+1} - 0.5 \max(0, \min(\alpha_R, \alpha_{R,L}, \beta_R)) \Delta \Phi_{i+\frac{3}{2}} \quad (2)$$

where β determines the type of limiting and α is the multi-dimensional restriction coefficient as follows.

$$\alpha^+ = g \left(2 \max(1, r_{i,j,k}^{-x}) \left[\frac{\left(1 + \frac{\tan \bar{\theta}_{i+1,j,k}}{r_{i+1,j,k}^{+x}} + \frac{1}{r_{i+1,j+1,k}^{+x} r_{i,j+1,k}^{+y} \cos \bar{\theta}_{i+1,j+1,k}} \frac{\tan \bar{\phi}_{i+1,j+1,k}}{\tan \bar{\theta}_{i,j,k}} \right)}{\left(1 + \tan \theta + \frac{\tan \phi}{\cos \theta} \right)} \right] \right) \quad (3)$$

$$\alpha^- = g \left(2 \max(1, r_{i+1,j,k}^{+x}) \left[\frac{\left(1 + \frac{\tan \bar{\theta}_{i+1,j,k}}{r_{i,j,k}^{-x}} + \frac{1}{r_{i,j+1,k}^{-x} r_{i,j,k}^{-y} \cos \bar{\theta}_{i+1,j+1,k}} \frac{\tan \bar{\phi}_{i+1,j+1,k}}{\tan \bar{\theta}_{i,j,k}} \right)}{\left(1 + \tan \theta + \frac{\tan \phi}{\cos \theta} \right)} \right] \right) \quad (4)$$

It is noted that α is a function of multi-dimensional flow parameters such as flow angle, cell-aspect ratio and local slopes.

NUMERICAL RESULTS

Here, we consider the three-dimensional normal shock discontinuity in order to investigate the shock-capturing characteristics of TVD MUSCL limiters and MLP. This test shows the advantages of MLP clearly in terms of monotonicity and convergence.

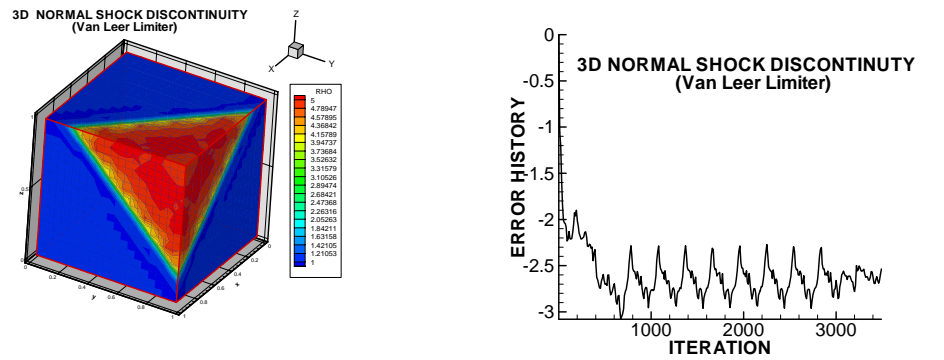


Figure 1. Density contour and error history : van leer limiter

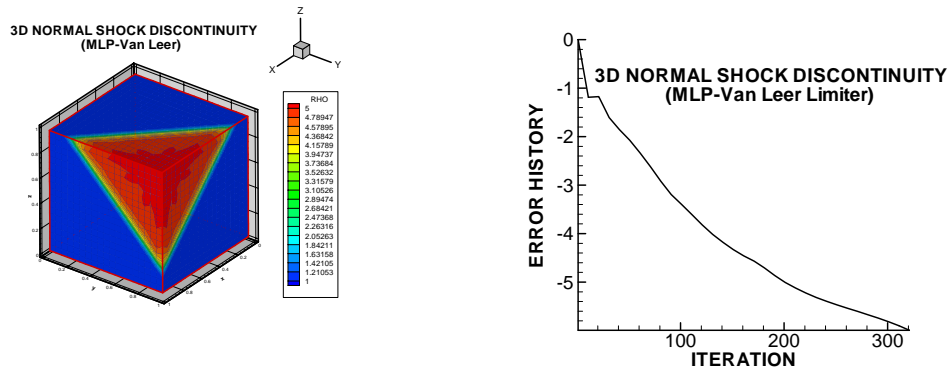


Figure 2. Density contour and error history : MLP-van leer limiter

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