A BDDC algorithm for mortar discretization

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ABSTRACT

A BDDC (balancing domain decomposition by constraints) algorithm is developed for elliptic problems with mortar discretizations for geometrically non-conforming partitions in both two and three spatial dimensions. The coarse component of the preconditioner is defined in terms of one mortar constraint for each edge/face which is an intersection of the boundaries of a pair of subdomains. A condition number bound of the form $C \max_i \{(1 + \log(H_i/h_i))^3\}$ is established. In geometrically conforming cases, the bound can be improved to $C \max_i \{(1 + \log(H_i/h_i))^2\}$. This estimate is also valid in the geometrically nonconforming case under an additional assumption on the ratio of mesh sizes and jumps of the coefficients. This BDDC preconditioner is also shown to be closely related to the Neumann-Dirichlet preconditioner for the FETI–DP algorithms of [2,3] and it is shown that the eigenvalues of the BDDC and FETI–DP methods are the same except possibly for an eigenvalue equal to 1.

INTRODUCTION

A BDDC algorithm was first introduced by Dohrmann [1] as an improvement of the balancing Neumann-Neumann method and using different coarse finite element spaces. The coarse space consists of a weighted sum of functions each of which minimizes the local discrete energy norm with certain constraints on the subdomain interfaces; continuity of the solutions at vertices, or average or momentum matching condition on solutions over edges/faces are considered in [1,5–7]. The resulting coarse problem then gives less local coupling between the subdomains than for the older balancing methods and more freedom in choosing the constraints to improve the convergence.

The constraints on the coarse finite element space are basically the same as those of a FETI–DP algorithm. In a FETI–DP algorithm, a linear system formulated for a set of dual variables is solved after eliminating the primal unknowns related to the primal constraints, given by average matching condition over edges/faces or continuity of the solutions at vertices. The resulting linear system, in itself, contains a coarse problem while its preconditioner is built only from subdomain problems. In a BDDC method, a linear system of the primal variables is solved iteratively with a preconditioner that has both coarse and subdomain components. This provides BDDC methods with more flexibility, allowing for the use of inexact coarse problems. Thus, an inexact coarse problem can be introduced by applying the BDDC method recursively to the coarse problem; see Tu [8].

Recently the BDDC methods have been shown to be closely related to the FETI–DP methods. A condition number bound of the BDDC operator was first given by Mandel and Dohrmann in [6]. They proved a $C(1+\log(H/h))^2$ bound that is comparable to that for the FETI–DP methods. Further, Mandel, Dohrmann, and Tezaur [7] showed that the eigenvalues of the FETI–DP and BDDC operators are the same except possibly for eigenvalues equal to 0 and 1. Recently, a new formulation of the BDDC method was given by Li and Widlund [5]. They introduced a change of variables as well as an average operator for the BDDC method based on the jump operator used in [4] in the analysis of FETI–DP methods.

In this presentation, we will describe a BDDC algorithm with a mortar discretization and a change of variables. Primal constraints on edges/faces are introduced. We consider quite general geometrically non-conforming partitions and the second generation of the mortar method as well as the dual basis mortar methods. A preconditioner is then proposed which uses a certain weight matrix D, that leads to the condition number bound: $C \max_i \{(1+\log(H_i/h_i))^3\}$. The algorithm can also be applied to a geometrically conforming partition and then gives a better bound: $C \max_i \{(1+\log(H_i/h_i))^2\}$. The same bound can be established for geometrically non-conforming partitions with an additional assumption on the mesh sizes.

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