A POSTERIORI ERROR ESTIMATES FOR THE FULLY DISCRETE MIXED METHODS FOR PARABOLIC PROBLEMS

Dongho KIM [1] and Eun-Jae PARK [2]

1) *Department of University College, Yonsei University, Seoul 120-749, KOREA*

2) *Department of Mathematics, Yonsei University, Seoul 120-749, KOREA*

Corresponding Author : Dongho KIM, dhkimm@yonsei.ac.kr

ABSTRACT

We construct a posteriori error estimator for the approximations to solutions of linear parabolic equations. We consider discretizations of the problem by backward Euler schemes in time and mixed finite element methods in space. Especially, mixed finite element spaces are permitted to change at different time levels in order to efficiently deal with problems with localized phenomena, such as shocks, sharp fronts or layers which move as time changes. Applying elliptic reconstruction idea, introduced in [1], to the mixed method, we derive a posteriori estimates which yields global upper bounds in time and space with the norms of $L_\infty(0,T;L_2(\Omega))$ and $L_2(0,T;L_2(\Omega))$ for the scalar function and the flux, respectively. We also derive a posteriori error estimates for time discretizations by modified Crank-Nicolson methods. By introducing proper Crank-Nicolson reconstructions corresponding to the mixed method, we get estimators having known rates of convergence in time.

INTRODUCTION

In the last 25-30 years adaptive procedures for the numerical solution of partial differential equations have received considerable attentions and are now standard tools in science and engineering. A posteriori error estimators are an essential ingredient of adaptivity. They are computed from the known values such as the given data of the problem and the computed numerical solutions; they provide quantitative estimates for the actual error and give base on adaptive mesh refinement strategy to optimize the computational work needed to reach a certain accuracy. Their usefulness is especially apparent when the exact solution has strong, geometrically localized, variations or exhibits singularities.

Few a posteriori estimates have been derived for parabolic problems in contrast with elliptic problems. In this paper we derive an a posteriori error estimator for backward Euler and modified Crank-Nicolson mixed finite element method for linear parabolic problems:

$$\begin{aligned} u_t - \operatorname{div}(A\nabla u) = f &\quad \text{in } \Omega \times (0,\mathrm{T}], \\ u = 0 &\quad \text{on } \partial\Omega \times (0,\mathrm{T}], \\ u(\cdot,0) = u_0 &\quad \text{in } \Omega. \end{aligned} \tag{1}$$

Parabolic problems might involve time changing localized phenomena such as initial transient, sharp fronts or layers. It is natural that mixed finite element spaces are permitted to change at different time levels. In deriving the estimates we use so called *elliptic reconstruction* technique of [1] for the model problem of semidiscrete finite element methods. Through this approach, we can use any available a posteriori estimates for elliptic equations to control the main part of the spatial error. In particular, in order to get estimators having known rates of convergence in time for Crank-Nicolson scheme, proper *Crank-Nicolson mixed reconstructions* is introduced.

REFERENCES

1. Makridakis, Ch. and Nochetto, R. H., ”Elliptic reconstruction and a posteriori error estimates for parabolic problems”, *SIAM J. Numer. Anal.* Vol. 41, no. 4, 2003, pp. 1585-1594.