

# Decision of Weights in the Discriminatory Processor Sharing System

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## ABSTRACT

In the discriminatory processor-sharing (DPS) system with a single processor and  $K$  job classes, all jobs present in the system are served simultaneously with rates controlled by a vector of weights  $\{g_j > 0; j = 1, \dots, K\}$ . When all  $g_i$  are equal, the DPS system reduces to the egalitarian processor-sharing (PS) system. In this paper we show how the weights of DPS must be chosen in order to make DPS outperform PS.

## INTRODUCTION

The discriminatory processor-sharing (DPS) has been of considerable interest and is useful in modelling some computer and communication systems. In the DPS discipline with a single processor and  $K$  job classes, all jobs present in the system are served simultaneously with rates controlled by a vector of weights  $\{g_j > 0; j = 1, \dots, K\}$ . If there are  $n_i$  class  $i$  jobs in the system,  $i = 1, \dots, K$ , then each class  $j$  job receives a fraction  $\frac{g_j}{\sum_{i=1}^K n_i g_i}$  of the processor capacity. When all  $g_i$  are equal, the DPS discipline reduces to the egalitarian processor-sharing (PS) discipline.

Using the DPS discipline it is possible to give preference to one or more classes of jobs at the expense of others. Thus, an appropriate choice of the DPS weights may enable differentiated quality of service among different job classes. The range of applications of the DPS is broad. The DPS may be an appropriate model as a fluid approximation of weighted round robin. The DPS provides a natural approach for modelling the flow-level performance of differentiated bandwidth-sharing mechanisms in networks, see for instance [1,3,4]. For example, Altman et al. [1] evaluated the performance of a common bottleneck shared by several TCP flows by using the DPS weights as the reciprocal of the round trip times.

## COMPARISON OF DPS AND PS

For the single-class queues, it is well known that giving preferential treatment to short jobs reduces the mean sojourn time of the system [6]. For the queue with multiple classes, Avrachenkov [2] proved that if the DPS system give preferential treatment to the class with smaller mean required service time (that is,  $\mu_1^{-1} \leq \mu_2^{-1} \leq \dots \leq \mu_K^{-1}$  and  $g_1 \geq g_2 \geq \dots \geq g_K$ ), then DPS outperforms PS, under the following condition:

**Condition 1.** There exists a  $j^*, j = 1, \dots, K$ , such that for all  $x \geq 0$ ,

$$\begin{cases} T_k^{DPS}(x) \leq T_k^{PS}(x) & \text{for all } k < j^*, \\ T_k^{DPS}(x) \geq T_k^{PS}(x) & \text{for all } k \geq j^*, \end{cases}$$

where  $T_k^{DPS}(x)$  and  $T_k^{PS}(x)$  denote the conditional mean sojourn time of a class  $k$  job whose required service time is  $x$  in the DPS and PS systems, respectively.

Avrachenkov et al. [2] reported that they did not find any example that Condition 1 does not hold under the case when  $\mu_1^{-1} \leq \mu_2^{-1} \leq \dots \leq \mu_K^{-1}$  and  $g_1 \geq g_2 \geq \dots \geq g_K$ .

Let  $\mathbf{N}^g = (N_1^g, \dots, N_K^g)$  and  $\mathbf{N}^{PS} = (N_1^{PS}, \dots, N_K^{PS})$  be the vectors where  $N_i^g$  and  $N_i^{PS}, i = 1, \dots, K$ , denote the number of class  $i$  jobs in the DPS and PS systems, respectively, at steady state. Let  $|\mathbf{N}^g| = N_1^g + \dots + N_K^g$  and  $|\mathbf{N}^{PS}| = N_1^{PS} + \dots + N_K^{PS}$  be the total number of jobs in the DPS and PS systems, respectively, at steady state.

In this work, we present an example that Condition 1 holds with  $\mu_1^{-1} \leq \mu_2^{-1} \leq \dots \leq \mu_K^{-1}$  and  $g_1 \geq g_2 \geq \dots \geq g_K$ . We also prove that DPS outperforms PS if  $g_1 \geq g_2 \geq \dots \geq g_K$  without Condition 1.

**Theorem 1** (1) If  $g_1 \geq g_2 \geq \dots \geq g_K$ , then  $\mathbb{E}[|\mathbf{N}^g|] \leq \mathbb{E}[|\mathbf{N}^{PS}|]$ .

(2) If  $g_1 \leq g_2 \leq \dots \leq g_K$ , then  $\mathbb{E}[|\mathbf{N}^g|] \geq \mathbb{E}[|\mathbf{N}^{PS}|]$ .

## COMPROMISE BETWEEN REDUCING MEAN SOJOURN TIME AND INCREASING FAIRNESS

It is observed that the mean sojourn time of DPS with large weight parameters on fast classes is less than that of DPS with large weight parameters on slow classes. On the other hand, DPS systems are fair in the sense that all classes have the same mean slowdown when all classes have the same weight parameters.

We introduce a cost function  $w_1\mathbb{E}[T_1] + w_2\mathbb{E}[(S - \frac{1}{1-\rho})^2]$  that measures weighted sum of mean sojourn time and a deviation of the slowdown. We find a way to calculate the first and the second moments of the slowdown for the M/M/1 DPS system and provide a method for choosing weight parameters that compromise unfairness with reducing mean sojourn time.

## REFERENCES

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