

Optimal Portfolio, Consumption-Leisure and Retirement Choice Problem with a Cobb-Douglas Utility and a CES Utility

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ABSTRACT

We study the optimal portfolio, consumption-leisure, and retirement choice of an infinitely-lived economic agent with a Cobb-Douglas utility and a CES (Constant Elasticity of Substitution) utility. Different from the previous works in investment problems, particular aspects of our problem are that the utility level of the agent is determined by both consumption and leisure and that the agent has a retirement option which can be exercised optimally considering trade-off between utility effect from leisure and wealth effect from labor. We show that the agent retires optimally if his wealth exceeds a certain critical level. A closed form solution can be provided by solving a free boundary value problem. In particular, the critical wealth level, the optimal consumption-leisure and portfolio policy are given in closed form.

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