

# Stability of fixed points on piecewise smooth systems

Younghae Do<sup>1</sup>

1) *Department of Mathematics, Kyungpook National University, Daegu 702-701, KOREA*

Corresponding Author : Younghae Do, yhdo@knu.ac.kr

## ABSTRACT

The stability type of fixed points of continuously differential dynamical systems can be determined by their Jacobian information. But in practical situation, we often meet piecewise smooth maps with a border separating two smooth regions. The discontinuous change in the Jacobian elements results in many atypical bifurcation phenomena, which is called border collision bifurcations, for instance a periodic orbit turning directly into a chaotic orbit, or multiple attractors coming into existence or going out of existence as the parameter is varied across some critical bifurcation value, etc. Recent research has shown the new type of border collision bifurcations, which is called the dangerous border collision bifurcations where a fixed point remains stable at both sides of the critical bifurcation point, but the orbit becomes unbounded at the critical point of bifurcation. Possibility for occurring such dangerous border collision bifurcations has been pointed out through numerical exploration but the mechanism causing its occurrence is not yet known. In the present paper, we focus on determining the stability type of fixed points of piecewise smooth systems. Specifically, consider non-differentiable fixed points which are located exactly on the border. In order to investigate the behavior of trajectories on piecewise linear systems having such fixed points, we consider the angle dynamics of the circle map defined on the unit circle with the assigned dilation ratios generated by the original piecewise linear system. From dynamical features of the circle map on the unit circle, we can measure the stability type of fixed points, which can be compared to the role of the magnitude of eigenvalues in continuously differential dynamical systems. Our results will help to understand the mechanism of such dangerous border collision bifurcations.

## REFERENCES

## REFERENCES

1. A. B. Nordmark, *J. Sound Vib.* **145**, 279-297 (1991).
2. S. Foale and S. R. Bishop, *Philos. Trans. R. Soc. London A* **338**, 547 (1992).
3. C. Budd and F. Dux, *Nonlinearity* **7**, 1191 (1994).
4. H. Lamba and C. Budd, *Phys. Rev. E* **50**, 84 (1994).
5. W. Chin, E. Ott, H. E. Nusse, and C. Grebogi, *Phys. Rev. E* **50**, 4427 (1994).
6. M. Ohnishi and N. Inaba, *IEEE Trans. Circuits Syst.* **41**, 433 (1994).
7. G. Yuan, S. Banerjee, E. Ott and J. A. Yorke, *IEEE Transactions on circuits and systems* **45**, 707-716 (1998).

8. H. E. Nusse, E. Ott, and J. A. Yorke, *Phys. Rev. E* **49**, 1073 (1994).
9. M. Garcia, A. Chatterjee, A. Ruina, and M. Coleman, *ASME J. Biomech. Eng.* **120**, 281 (1998).
10. J. Sun, F. Amellal, L. Glass, and J. Billette, *J. Theor. Biol.* **173**, 79 (1995).
11. H. E. Nusse and J. A. Yorke, *Physica D* **57**, 39-57 (1992).
12. M. di Bernardo, M. I. Feigin, S. J. Hogan and M. E. Homer, *Chaos Solitons Fractals* **10**, 1881 (1999).
13. M. A. Hassouneh, E. H. Abed and H. E. Nusse, *Phy. Rev. Lett.* **92**, 70201(2004).
14. Y. Do and H. Baek, *submitted*.
15. J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer-Verlag, 1983.
16. K. T. Alligood, T. D. Sauer, and J. A. Yorke, *Chaos An introduction to Dynamical systems*, Springer-Verlag, 1996.