Large-update interior point algorithm for LCP

Gyeong-mi Cho [1] , Min-kyung Kim [2]

1) *Department of Multimedia Engineering, Dongseo University, Pusan 617-716, KOREA*

2) *Department of Mathematics, Pusan National University, Pusan 609-735, KOREA*

Corresponding Author : Gyeong-mi Cho, gcho@dongsea.ac.kr

ABSTRACT

In this paper we propose a new large-update primal-dual interior point algorithm for $P_*(\kappa)$ linear complementarity problems (LCPs). We generalize the analysis of BER's primal-dual interior point algorithm for LP to $P_*(\kappa)$ LCPs. New search directions and proximity measures are proposed based on a new kernel function which has linear growth term. We showed that if a strictly feasible starting point is available, then the new large-update primal-dual algorithms for solving $P_*(\kappa)$ LCPs have the similar polynomial complexity for LO which is the same complexity with the large-update primal-dual Newton method for $P_*(\kappa)$ LCPs.

INTRODUCTION

In this paper we consider the linear complementarity problem (LCP) as follows :

$$\begin{cases} s = Mx + q, \\ xs = 0, \\ x \geq 0, \;\; s \geq 0, \end{cases} \tag{LCP}$$

where $M \in R^{n\times n}$ is a $P_*(\kappa)$ matrix and $q \in R^n$.

The primal-dual interior point method for linear programming was first introduced in [3] and [7]. Kojima et al. [3] first proved the polynomial computational complexity of the algorithm for LP, and since then many other algorithms have been developed based on the primal-dual strategy. Kojima, Mizuno and Yoshise ([5]) proposed a polynomial time algorithm for monotone linear complementarity problems. They also proposed an $O(\sqrt{n}L)$ potential reduction algorithm ([6]). The existence of a central path is very important for interior point algorithms. Kojima et al. [4] proved the existence of the central path for any $P_*(\kappa)$ LCP and generalized the primal-dual interior point algorithm to $P_*(\kappa)$ LCP and they established the same complexity results. Since then a variant of an interior point algorithm's quality is measured by the fact whether it can be generalized to $P_*(\kappa)$ LCPs or not.([2]). Miao ([8]) extended the MTY predictor-corrector method for $P_*(\kappa)$ LCPs. His algorithm uses the l_2 neighborhood of the central path and has $O((1+\kappa)\sqrt{n}L)$ iteration complexity. Recently, Illés and Nagy [2] give a version of the Mizuno-Todd-Ye predictor-corrector interior point algorithm for the $P_*(\kappa)$-matrix linear complementarity problem and show that the complexity of the algorithm is $O((1+\kappa)^{\frac{3}{2}}\sqrt{n}L)$. They choose τ and τ' neighborhood parameters in such a way that a predictor step following by one corrector step at each iteration. For larger value of κ the values of τ and $\tau^{'}$ are fastly decreasing, therefore the constant in the complexity result is increasing.

Most of polynomial-time interior point algorithms for LO are based on the use of the logarithmic barrier function. Peng et al.([9]) introduced self-regular barrier functions for primal-dual interior-point methods for linear optimization and proved the best complexity for large-update primal-dual interior point methods for LO with some specific self regular barrier function. Recently Roos et al. [1] proposed a new primal-dual interior point method for LO based on a new proximity function which has linear growth term.

In this paper we propose a new large-update primal-dual interior point method which generalize BER's algorithm for LO to $P_*(\kappa)$ LCP and get the similar iteration complexity $O((1+2\kappa)qnlog\frac{n}{\varepsilon})$ for $P_*(\kappa)$-matrix linear complementarity problem. At each iteration, our algorithm selects a target on the central path with a large-update from the the current iterate, and then new search direction is used based on a kernel function which is different from the ones in [2], [4], [8], [9]. Then we choose the largest possible step size before leaving a neighborhood in which the proximity measure is given by the kernel function. Our neighborhood is larger than l_2-neighborhood in [8] and our analysis is simpler than the one in [2]. The Algorithm obtains the same complexity with the classical large-update log-barrier interior point method.

REFERENCES

1. Bai, Y.Q. and Roos, C., *A primal-dual interior-point method based on a new kernel function with linear growth rate*, Proceedings Perth Industrial Optimization meeting, 2002.
2. Illes, T. and Nagy, M., *The Mizuno-Todd-Ye predictor-corrector algorithm for sufficient matrix linear complementarity problem*, Eötvös Loránd University of Sciences, 2004.
3. Kojima, M., Megiddo, N., Noma, T., and Yoshise, A., *A primal-dual interior point algorithm for linear programming*, in: Progress in Mathematical Programming; Interior Point Related Methods, (ed N. Megiddo), Springer Verlag, New York, 1989, 29-47.
4. Kojima, M., Megiddo, N., Noma, T., and Yoshise, A., *A Unifided Approach to Interior Point Algorithms for Linear Complementarity Problems*, Volume 538 of lecture Notes in Computer Science, Springer-Verlag, Berlin, Germany, 1991.
5. Kojima, M., Mizuno, S., and Yoshise, A., *A polynomial-time algorithm for a class of linear complementarity problems*, Mathematical Programming, 44(1989) 1-26.
6. Kojima, M., Mizuno, S., Yoshise, A., *An $O(\sqrt{n}L)$ iteration potential reduction algorithm for linear complementarity problems*, Mathematical Programming 50(1991) 331-342.
7. Megiddo, N., *Pathways to the optimal set in linear programming*, in (ed N. Megiddo) Progress in Mathematical Programming ; Interior point and Related Methods, Springer Verlag, New York, 1989, 313-358.
8. Miao, J., *A quadratically convergent $O((1+\kappa)\sqrt{n}L)$-iteration algorithm for the $P_*(\kappa)$-matrix linear complementarity problem* , Mathematical Programming 69(1995) 355-368.
9. Peng, J., Roos, C., and Terlaky, T., *A new class of polynomial primal-dual methods for linear and semidefinite optimization*, European Journal of operations Research, 143(2)(2002) 234-256.