CLIMATE MODELING: A CHALLENGE FOR MATHEMATICIANS

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ABSTRACT

We present here a master framework (or paradigm) for climate modeling of the Earth. This framework covers the full range of climate phenomena, and it can be suitably adapted and simplified in order to study various climatic features on any prescribed long term time scale. Since we identify the climate of the Earth with the longtime dynamics of the environment, this paper focuses on the dynamics of oceanic flows. We consider the oceanic flow with an infinitesimal interface model, which aggregates the net effects of all the relevant atmospheric activity at the surface of the ocean.

The master framework we present here comes with a rigorous mathematical foundation. In order to derive an hierarchy of good climate models, we begin with the concept of an allowable perturbation of a given (good) model. Such perturbations have the property that they preserve much of the longtime dynamical information of the original model, see Section 6.

The role of the planetary motion in this paradigm is to introduce a time-dependent gravitational field which acts on the oceans of the Earth. Based on good physics (including several centuries of calculations of the planetary motion in the solar system) and good mathematics (including the KAM theory), we adopt the Quasi-Periodic Ansatz (QPA), which states that the gravitational field, as well as, the heating and cooling of the surface of the ocean, are quasi-periodic functions of time. By using the QPA, we introduce the method of Partial Averaging (PA). The PA method is a rigorous tool which permits us to separate climatic phenomena by their time scales by using allowable perturbations. We give two applications of the PA method: (1) by replacing the the daily heating and cooling of the ocean surface with its partial average, we obtain an allowable perturbation; and (2) we use the PA method to construct an El Niño model based on the gravitational field of the SEM (Sun-Earth-Moon) system.

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