Some Fundamental Problems in Robotics: a Geometric Perspective

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ABSTRACT

We review some fundamental problems that arise in robot mechanics, and show how tools from differential geometry and Lie group theory have played an integral part in their solution.

INTRODUCTION

From its initial promise in the 1960's and 1970's, to its premature demise in the late 1980's and 1990's, robotics today is now witnessing a tremendous resurgence of interest. This revival can be traced in part to the recent progress made in the development of robotic platforms, with the advent of humanoid robots that can walk, climb stairs, even dance. At the same time, field and service robots are now routinely being deployed in automation, construction, rescue, and other diverse applications, and the technology has now become available for hobbyists to construct and program robots using off-the-shelf components and general programming tools.

This rejuvenation of robotics has been driven not only by hardware and computing advances, but also by a better understanding and solution of the fundamental problems in robotics. The theoretical underpinnings of a robotics science are now in place, and it is now possible to frame some of the fundamental questions in robotics using the language of modern mathematics. For the aspects of robotics related to mechanics, it turns out that differential geometry is a natural setting in which to formulate the fundamental questions and their solutions.

In this talk we focus on three aspects of robot mechanics in which tools from differential geometry and Lie groups have played a central role: the question of how to design kinematic chains for a given set of tasks, deriving the equations of motion for complex robotic systems, and generating natural robotic movements, in the sense of minimizing certain physical criteria and emulating the movements of humans and animals.

OPTIMAL KINEMATIC DESIGN AND THE THEORY OF HARMONIC MAPS

Most of the recent efforts in kinematic design center around identifying and quantifying certain qualitative features of robot performance; these measures are then optimized to arrive at some form of "best" design. Aspects of workspace volume, dexterity, and dynamic response all have been studied extensively in the literature. In fact, upon closer inspection one is struck by the variety of ways in which robot performance can be defined and measured. These measures are usually formulated in terms of local coordinate representations for the robot's kinematics and dynamics, which are generally accepted without question. However, one must be careful to formulate the measure in a way that reflects the intrinsic properties of the mechanism, rather than that of the particular choice of coordinates. Complicating this further is the fact that neither the space of rigid-body motions, nor the configuration space of most mechanisms containing closed loops, is a vector space. Clearly any meaningful performance measure should be formulated such that it is independent of the choice of coordinates, and takes into account the nonlinearity of both the configuration space and the rigid-body motions.

One of the viewpoints advanced here is the use of global, coordinate-free methods of differential geometry to characterize robot performance. The global kinematic and dynamic properties of a robotic mechanism can be represented quite naturally within this geometric framework, as the recent reworking of the classical screw theory of rigid body motions in terms of the one parameter subgroups of the Euclidean group attests; the entire body of results from Lie theory now becomes available for kinematic analysis. Similarly, many of the existing robot performance measures can be described mathematically in a much simpler fashion by ideas that were developed in a completely general geometric setting. One result is that their physical meaning now becomes more apparent. For example, the integral functional of *harmonic mapping* theory leads to a simple formulation of global dexterity, and workspace volume becomes a straightforward application of volume measures on Riemannian manifolds.

LIE GROUP FORMULATION OF ROBOT DYNAMICS

A second domain in which geometric tools have been particularly helpful is in the formulation of the equations of motion for general kinematic chains. Although from a certain point of view generating the equations of motion for robots, and multibody systems in general, presents no theoretical difficulty, in practice the symbolic and computational complexity of the ensuing equations has lead to repeated attempts in the past to formulate the equations in a more compact, computationally efficient manner.

Using standard ideas from Lie groups and Riemannian geometry, we present a formulation of the dyamic equations for rigid multibody systems that leads to a particularly simple set of closed-form equations. In particular, the inertia matrix can be explicitly factored into terms involving kinematic and inertial parameters, respectively, which can in turn be easily differentiated with respect to any of these parameters. The differential geometric approach permits a high-level, coordinate-free view of robot dynamics that shows explicitly some of the connections with the larger body of work in mathematics and physics. The geometric formulation is also attractive for applications like robot design and calibration, motion optimization, and optimal control, in which analytic gradients involving the dynamic equations are usually required for obtaining numerically stable solutions in reasonable time.

GENERATING OPTIMAL ROBOT MOVEMENTS

As a third and final application, that draws upon the previous geometric formulations of kinematics and dynamics, we consider the problem of generating optimal motions for robots. Among the many innate physical abilities of humans, motor control is the skill that is most often taken for granted, as it seems to require very little conscious effort on our part. Only when a particular motor skill is impaired or lost does one then begin to fully appreciate the difficulty of motor control. It comes as no surprise that these exact same difficulties are encountered, indeed even magnified, when attempting to endow robots with a movement generation capability like that of humans.

Our broad aim here is to emulate the low-level capabilities of human motor coordination and learning within the framework of optimal control theory. Our approach is based on the simple observation that, in nearly all of the motor learning scenarios that we have observed, some form of optimization with respect to a physical criterion is taking place. Moreover there is ample biological evidence to justify an optimization-based approach to movement generation; in the literature one can find many optimal control-based studies of various human motions, ranging from maximum-height jumping, voluntary arm movements, maintaining postural balance, minimum-time running and swimming, even wheelchair propelling. Besides some of the more obvious optimization criteria like minimum energy or control effort, strategies that involve minimizing the derivative of acceleration (or jerk), as well as muscle or metabolic energy costs, have also been examined in the context of specific arm motions.

Some researchers have also presented biological evidence suggesting that the nervous system implicitly performs inverse dynamics to generate feedforward motor commands, particularly for fast motions. Previous research also shows that it is possible to identify accurate internal models from movement data, and that such strategies can be successfully implemented in robots. Aproaches to motor coordination and learning based on equilibrium and hierarchical approaches inspired by biological systems, and dynamical systems theory, have also been presented.

From an engineering perspective an optimization-based approach to movement generation usually strikes one as the first reasonable thing to try. The reason that such approaches have been largely unsuccessful, it seems, is that the complexity of the dynamic equations inevitably lead to intractable optimization problems. Indeed, the intractability of the optimization seems at least partly—if not largely—responsible for the recent flurry of attention given to, *e.g.,* neural networks, genetic algorithms, and other evolutionary optimization approaches to motor learning.

One of the arguments put forth here is that movement generation based on dynamic models and classical descent-type optimization methods is indeed a computationally feasible paradigm. Aside from the complexity of the nonlinear dynamics, another reason classical descent methods, despite their reliability (indeed, in many cases these algorithms are the only ones that can guarantee local optimality and convergence), are bypassed in many of today's motion learning schemes is their reliance on gradient and Hessian information. Although in principle one can numerically approximate these quantities, for problems involving even moderately complex multibody systems, approximated gradients and Hessians more often than not lead to ill-conditioning, instability, and poor convergence behavior, not to mention a significant increase in computation.

By appealing to our earlier Lie group formulation of the equations of motion, it is now possible to develop algorithms that render the optimization problem tractable. In many cases the optimized motions can even be obtained quite efficiently and in a numerically robust way. The key lies in the ability to recursively evaluate the nonlinear dynamics, and also to recursively compute exact analytic gradients and Hessians without resorting to numerical approximations. The resulting algorithms are still computation-intensive by today's standards, but are $O(n)$ with respect to the number of rigid bodies comprising the system, and perhaps most important of all, robust.

REFERENCES

1. Asada, H 1983. A geometrical representation of manipulator dynamics and its application to arm design. *Trans. ASME J. Dyn. Sys., Meas., and Contr.* 105(3):131-135.
2. Brockett, R. W. Robotic manipulators and the product of exponentials formula. *Proc. Symp. Math. Theory Networks and Systems*. Beer Sheba, Israel, pp. 120-129.
3. Eells, J., and Sampson, J.H. 1964. Harmonic mappings of Riemannian manifolds. *Amer. J. Math.*, vol. 86, pp. 109-160.
4. Eells, J., and Lemaire, L. 1978. A report on harmonic maps. *Bulletin London Math. Soc.*, vol. 10, pp. 1-68.
5. Gupta, K. C. 1986. On the nature of robot workspace. *Int. J. Robotics Research*, 5(2):112-121.
6. Hollerbach, J. M. 1985. Optimum kinematic design of a seven degree of freedom manipulator. In *Robotics Research: The Second International Symposium*. H. Hanafusa and H. Inoue, eds. Cambridge: MIT Press.
7. Loncaric, J. 1985. *Geometric Analysis of Compliant Mechanisms in Robotics*. PhD Thesis, Harvard University, 1985.
8. Paden, B., and Sastry, S. 1988. Optimal kinematic design of 6R manipulators. *Int. J. Robotics Research* 7(2).
9. Park, F. C., and Brockett, R. W. 1994. Kinematic dexterity of robotic mechanisms. *Int. J. Robotics Research* 13(2):1-15.
10. Roth, B. 1976. Performance evaluation of manipulators from a kinematic viewpoint. National Bureau of Standards, NBS Special Publication 495, pp. 39-61.
11. Salisbury, J. K., and Craig, J. J. 1982. Articulated hands: force control and kinematic issues. *Int. J. Robotics Research*, 1(1):4-17.
12. Sordalen, O. J., Nakamura, Y., and Chung, W.J. 1994. Design of a nonholonomic manipulator. *Proc. IEEE Int. Conf. Robotics Autom.* pp. 8-13.
13. Spong, M. W. 1992. Remarks on robot dynamics: canonical transformations and riemannian geometry. *Proc. IEEE Int. Conf. Robotics Autom.* pp. 454-472.
14. Yoshikawa, T. 1985. Manipulability of robotic mechanisms. *Int. J. Robotics Research* 4(2):3-9.
15. Yoshikawa, T. 1985. Dynamic manipulability of robot manipulators. *Proc. IEEE Int. Conf. Robotics and Autom.*,, pp. 1033-1038
16. R. M. Alexander, “A minimum energy cost hypothesis for human arm trajectories,” *Biol. Cybern.*, vol. 76, pp. 97-105, 1997.
17. C. G. Atkeson, “Learning arm kinematics and dynamics,” *Ann. Rev. Neurosci.*, vol. 12, pp. 157-183, 1989.
18. J.T. Betts, “Survey of Numerical Methods for Trajectory Optimization,” *Journal of Guidance, Control and Dynamics*, vol. 21, no. 2, 193-207, 1999.
19. L.S. Crawford and S.S. Sastry, “Biological motor control approaches for a planar diver,” in *Proc. 34th IEEE Conf. Dec. Contr.*, New Orleans, LA, 1995, vol 4, pp. 3881-3886.
20. A. C. Fang and N. Pollard, “Efficient computation of optimal, physically valid motions,” *J. Robotics Society of Japan*, vol. 22, no. 2, pp. 23-27, 2004.

21. G. J. Garvin, M. Zefran, E. A. Henis, and V. Kumar, “Two-arm trajectory planning in a manipulation task,” *Biol. Cybern.*, vol. 76, pp. 53-62, 1997.

22. G. L. Gottlieb, D. M. Corcos, and G. C. Agarwal, “Strategies for the control of voluntary movements with one mechanical degree of freedom,” *Behavioral and Brain Sci.*, vol. 12, pp. 189-250, 1989.

23. A. Ijspeert, J. Nakanishi, and S. Schaal, “Movement imitation with nonlinear dynamical systems in humanoid robots,” *P*roc. IEEE Int. Conf. Robotics & Autom.

24. A. Karniel and G. F. Inbar, “A model for learning human reaching movements,” *Biol. Cybern.*, vol. 77, pp. 173-185, 1997.

25. Junggon Kim, Jonghyun Baek, and F. C. Park, “Newton-type algorithms for robot motion optimization,” *Proc. IEEE Int. Conf. Intelligent Robots & Systems*, Kyongju, Korea, pp. 1842-1847, 1999.

26. A. Kuo, “Optimal control model for analyzing human postural balance,” *IEEE Transactions on Biomedical Engineering*, vol. 42, no. 1, pp. 87-101, 1995.

27. N. Lan and P. E. Patrick, “Optimal control of antagonistic muscle stiffness during voluntary movements,” *Biological Cybernetics*, vol. 71, no. 2, pp. 123-135, 1994.

28. R. Maronski, “Minimum-time running and swimming: an optimal control approach,” *Journal of Biomechanics*, vol. 29, no. 2, pp. 245-249, 1996.

29. B. Martin and J.E. Bobrow, “Minimum Effort Motions for Open Chained Manipulators with Task-Dependent End-Effector Constraints,” *International Journal of Robotics Research*, Vol. 18, No. 2, February, 1999, pp. 213-224.

30. F. A. Mussa-Ivaldi, “Nonlinear force fields: a distributed system of control primitives for representing and learning movements,” *Proc. IEEE Int. Symp. Computational Intelligence in Robotics and Autom.*, pp. 84-90, 1997.

31. R.M. Murray, Z.X. Li, and S.S. Sastry, *A Mathematical Introduction to Robotic Manipulation.* Boca-Raton: CRC Press, 1994.

32. M. G. Pandy, F. E. Zajac, E. Sim, and W. S. Levine, “An optimal control model for maximum-height human jumping,” *J. Biomechanics*, vol. 23, no. 12, pp. 1185-1198, 1990.

33. N. Ogihara and N. Yamazaki, “Generation of human reaching movement using a recurrent neural network model,” *IEEE International Conference on Systems, Man, and Cybernetics*, vol. 2, pp. 692-697, 1999.

34. F. C. Park, J. E. Bobrow, and S. R. Ploen, “A Lie group formulation of robot dynamics,” *International Journal of Robotics Research*, vol. 14, no. 6, pp. 609-618, December 1995.

35. F. C. Park and Jinwook Kim, “Singularity analysis of closed kinematic chains,” *ASME J. Mechanical Design*, vol. 121, no. 1, pp. 32-38, 1999.

36. F. C. Park, Jihyeon Choi, and S. R. Ploen, “Symbolic formulation of closed chain dynamics in independent coordinates,” *Journal of Mechanism and Machine Theory*, vol. 34, no. 5, pp. 731-751, July 1999.

37. S. R. Ploen and F. C. Park, “Coordinate-invariant algorithms for robot dynamics,” *IEEE Transactions on Robotics and Automation*, vol. 15, no. 6, pp. 1130-1135, December 1999.

38. D. H. Rao and H. V. Kamat, “Artificial neural network for the emulation of human locomotion patterns,” *Engineering in Medicine and Biology Society*, IEEE , vol. 2, pp.80-81, 1995.

39. N. Schweighofer, M.A. Arbib, and M. Kawato, “Role of the cerebellum in reaching movements in humans. I. Distributed inverse dynamics control,” *European J. Neuroscience*, vol. 10, pp. 86-94, 1998.

40. S. Stroeve, “Learning combined feedback and feedforward control of a musculoskeletal system,” *Biol. Cybern.*, vol. 75, pp. 73-83, 1996.

41. J. C. W. Sullivan and A. G. Pipe, “An evolutionary optimisation approach to motor learning with first results of an application to robot manipulators,” *IEEE International Conference on Computational Cybernetics and Simulation*, vol. 5, pp. 4406-4411, 1997.

42. Y. Ting, P. N. Sheth, and C. E. Brubaker, “Application of energy optimal control to muscular force distribution for wheelchair propulsion,” *ASME Bioeng. Div. Publ. Bed.*, ASME, New York, vol. 20, pp. 517-520, 1991.

43. C.-Y. E. Wang, W. K. Timoszyk, and J. E. Bobrow, “Payload maximization for open chained manipulators: finding weightlifting motions for a Puma 762 robot,” *IEEE Trans. Robotics Autom.*, vol. 17, no. 2, April 2001.