Mathematical Modeling of Rayleigh Fading channels based on Markov Chains

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ABSTRACT

To mathematically model the Rayleigh fading channel, we propose a new method to partition the received Signal-to-Noise ratio (SNR) into a finite number of state and construct a finite state Markov chain. Computer simulations are performed to verify the accuracy of the model.

INTRODUCTION

Clark has proposed a statistical model for the received signal amplitude of the flat-fading channel based on scattering propagation [5]. Recently, finite-state Markov chains have been proposed to model the flat-fading channel, with states representing discrete, non-overlapping intervals of the received signal's power. This model is called the FSMC(Finite State Markov Chain) model. And, it is analytically tractable and closed-form results can be easily obtained with the model. So, it is widely used to study the fading channel.

In this thesis, we will describe how to model the Rayleigh Fading Channel where the signal power is according to the probability density function (PDF)

$$p_\gamma(\gamma) = \frac{1}{\gamma_0} exp(-\frac{\gamma}{\gamma_0}),\ \gamma \geq 0 \tag{1}$$

where γ_0 is the average SNR.

A convenient way of characterizing a mobile-radio communication channel is to use its baseband equivalent model. The relation between real channel impulse response,$h(t)$, and baseband equivalent model,$h_l(t)$, is the following :

$$h(t) = 2Re\{(h_l(t)exp(j2\pi f_c t)\} \tag{2}$$

where f_c is the carrier frequency.

If $u(t)$ and $v(t)$ are the baseband equivalent models of the input siganal,$s(t)$, and the output signal,$r(t)$, we can get the following equations.

$$v(t) = u(t) * h_l(t) \tag{3}$$

$$r(t) = Re\{(u(t) * h_l(t))exp(j2\pi f_c t)\} \tag{4}$$

where $*$ is the convolution operation. Since we assume the channel as flat fading, the equation (3) can be changed as the discrete-time baseband model,

$$v[m] = u[m]h_l[m]. \quad (5)$$

The above equation is induced by the assumption that symbol interval time is much larger than delay spread. $x(t)$ and $y(t)$, which are the real part and the imaginary part of the $h_l(t)$ respectively, are called as the In-phase component and the Quadrature-phase component. We know that the two fading components $x(t)$ and $y(t)$ of the Rayleigh fading channel are two independent Gaussian processes with zero mean and the following autocorrelation function.

$$R_X(\tau) = E[x(t)x(t+\tau)] = \frac{\gamma_0}{2} J_0(2\pi fm \cdot \tau), \quad (6)$$

$$R_Y(\tau) = E[y(t)y(t+\tau)] = \frac{\gamma_0}{2} J_0(2\pi fm \cdot \tau), \quad (7)$$

where $J_0(.)$ is the Bessel function of the first kind of order zero [1].
In this thesis, the new SNR partition method of the FSMC model is addressed for Rayleigh fading channel model. And then this model is compared with a computer simulation. We use the matlab's simulink for generating the channel gain $h_l[m]$.

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