

탄성체 내에서 발생하는 무판막펌핑에 관한 시뮬레이션

SIMULATIONS OF VALVELESS PUMPING IN AN OPEN ELASTIC TUBE

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ABSTRACT

Mathematical models and numerical simulations of flows driven by pumping without valves (valveless pumping) are presented. This work has been originally motivated by a biomedical objective to explain the complicated valveless blood flow mechanism in the circulation. For instance, the fetal circulation before the formation of valves and blood circulation during cardiopulmonary resuscitation. A mathematical model of valveless pumping either in a closed loop system or in an open system consists of a couple of tubes with different elasticities or radii. In this work, we develop new models which are not necessarily connected by two different materials. Although only one soft material is used or the soft and rigid materials are not connected, we have observed the existence of a net flow driven by the periodic compress-and-release action and the important features of valveless pumping that have been observed in earlier models or experiments. Our important result is that the direction and the magnitude of a net flow can be explained by the sign and the amount of power, which is work done on the fluid by the fluid pressure and the elastic wall over one period, respectively. A new feature in this work is that only one elastic material is sufficient to show the existence of a net flow in a valveless pump system. Because of a simple structure in our new model, it is much easier to construct a valveless pump system applicable to real world applications, such as a microelectromechanical system device.

INTRODUCTION

One can use the term valveless pumping for a fluid-structure mechanical system without valves driven by periodic pumping. In recent decades a great deal of interest in theoretical, computational, and experimental valveless pumping models have been increased.

The mechanism of valveless pumping can be observed in many biological systems, such as the circulations in the human fetus before the formation of valves [1] and other simple life organisms which do not have valves. A valveless pump system might also explain the thoracic pump mechanism during cardiopulmonary resuscitation (CPR). In the thoracic pump theory, it has been reported that the blood circulation in the cardiovascular system might exist even though the role of the heart is passive like other vessels and the cardiac valves do not function normally [2, 3, 4, 5].

In this paper, we have developed new mathematical models of valveless pumping using only one elastic open tube. In earlier models and experiments of valveless pumping, smoothly joined tubes with either different material or different geometrical properties have been used in a closed loop system or in an open system. In this work, we develop new models which are not necessarily connected by two different materials. In our models, the periodic compress-and-release action is applied to an asymmetric location of the elastic part. Although this model is unlike other earlier models, we have been able to observe the important characteristics of a valveless pump system. We have studied nine special cases including positive, almost zero, and negative net flows. Flowmeters, the motions of waves along the elastic tube, and the flow and pressure changes over one cycle are investigated in order to explain the complicated fluid dynamics in a valveless pump system.

MATHEMATICAL MODEL

Consider a viscous incompressible fluid which fills a periodic rectangular box containing an elastic open tube. Figure 1 displays the initial configuration of our two-dimensional model. In Figure 1, an open elastic tube (thin lines) and fluid markers (dots) are displayed. The rectangular box represents a computational domain. The fluid motions are driven by the periodic vertical compressand-release action on the left second quarter of the elastic tube (the pumping location is marked with the upper and lower arrows) in most of our simulations.

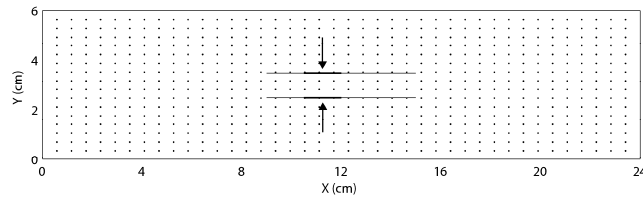


FIG 1. Initial configuration

To model the elastic tube, we used a mathematical and numerical method, the immersed boundary method. The immersed boundary method is applicable to the problems involving an elastic structure interacting with a viscous incompressible fluid. It has been applied to a variety of problems, particularly in biophysics, including simulations of blood flow in the heart [6, 7, 8, 9], the design of prosthetic cardiac valves [10], platelet aggregation during blood clotting [11], wave propagation in the cochlea [12], the flow of suspensions [13], peristaltic pumping of solid particles [14], aquatic animal locomotion [15], valveless pumping in a closed loop system [16, 17], whirling instability [18], and filaments in a flowing soap film [19]. The philosophy of the immersed boundary method is that the elastic material is treated as a part of the fluid in which singular forces are applied. The fluid and the elastic immersed boundary constitute a coupled mechanical system. The motion of the fluid is influenced by the force generated by the immersed boundary exerted on the fluid, but, at the same time, the immersed boundary moves at the local fluid velocity. The strength of this method is that it can handle the complicated and time dependent geometry of the elastic immersed boundary which interacts with the fluid, and that it does so while using a fixed regular lattice for the fluid computation.

We shall now describe the mathematical formulation of the equations of motion for the fluid-elastic boundary system:

$$\rho \left(\frac{\partial \mathbf{u}(\mathbf{x}, t)}{\partial t} + (\mathbf{u}(\mathbf{x}, t) \cdot \nabla) \mathbf{u}(\mathbf{x}, t) \right) + \nabla p(\mathbf{x}, t) = \mu \nabla^2 \mathbf{u}(\mathbf{x}, t) + \mathbf{f}(\mathbf{x}, t), \quad (1)$$

$$\nabla \cdot \mathbf{u}(\mathbf{x}, t) = 0, \quad (2)$$

$$\mathbf{f}(\mathbf{x}, t) = \int \mathbf{F}(s, t) \delta^2(\mathbf{x} - \mathbf{X}(s, t)) ds, \quad (3)$$

$$\frac{\partial \mathbf{X}(s, t)}{\partial t} = \mathbf{U}(s, t) = \int \mathbf{u}(\mathbf{x}, t) \delta^2(\mathbf{x} - \mathbf{X}(s, t)) d\mathbf{x}, \quad (4)$$

$$\mathbf{F}(s, t) = -\kappa_t (\mathbf{X}(s, t) - \mathbf{Z}(s, t)) + \kappa_c \left(\frac{\partial^2 \mathbf{X}(s, t)}{\partial s^2} \right). \quad (5)$$

The fluid and immersed boundary variables are written in lowercase and uppercase letters, respectively. Equations (1) and (2) are the viscous incompressible Navier–Stokes equations for the fluid in Eulerian form. The fluid velocity $\mathbf{u}(\mathbf{x}, t)$, fluid pressure $p(\mathbf{x}, t)$, and singular force density $\mathbf{F}(\mathbf{x}, t)$ are unknown functions of (\mathbf{x}, t) , where $\mathbf{x}=(x, y)$ are fixed Cartesian coordinates and t is time. The constant parameters ρ and μ are the fluid density and viscosity, respectively. The parameter κ_t is a stiffness constant between the physical boundary and the target positions, and κ_c is a stiffness constant of a linear spring between two adjacent boundary points. Equations (3) and (4) are the interaction equations between fluid and elastic boundaries. They convert from Lagrangian to Eulerian (3) and vice versa (4) by the two-dimensional Dirac delta function. Equation (3) defines the fluid force density by the integral transformation of the boundary force density. Equation (4) is, in fact, the no-slip condition, which states that the immersed boundary moves at the local fluid velocity. Both equations are in the integral form with the kernel $\delta^2(\mathbf{x} - \mathbf{X}(s, t))$. The kernel is a product of two one-dimensional Dirac delta functions: $\delta^2(\mathbf{x}) = \delta(x)\delta(y)$. Therefore, $\mathbf{f}(\mathbf{x}, t)$ is a singular force density. Equation (2.5) describes the boundary force density $\mathbf{F}(s, t)$ in Lagrangian form. $\mathbf{X}(s, t)$ is the immersed boundary, where $0 \leq s \leq L$, the unstressed lengths of the tube boundaries. A fixed value of the Lagrangian parameter s marks a material point of the immersed boundary. The boundary force is computed as a sum of two terms in (5). In the first term, the given function $\mathbf{Z}(s, t)$ is called the target position of the immersed boundary, and the details of its description are given later. This first term provides a restoring force that keeps the boundary points near their target positions. The second term in (5) models an elastic membrane under tension. Together, the two terms model the boundaries as tethered elastic membranes. Numerical and physical parameters in centimeter-gram-second units are given in Tables 1 and 2.

▪ TABLE 1. Computational parameters.

Parameters	Notation	Values
Fluid lattice	$N_x \times N_y$	512×128
Number of upper boundary points	M_1	513
Number of lower boundary points	M_2	513
Mesh width	$h = \Delta x = \Delta y$	0.0469 cm
Initial distance between boundary points	Δs	$h/4 = 0.0117$ cm
Time step	Δt	$0.5h^2 = 0.0011$ sec
Simulated time	t_{\max}	50 sec

▪ TABLE 2. Physical parameters.

Parameters	Notation	Values
Computational domain	$X_{\text{scale}} \times Y_{\text{scale}}$	24 cm \times 6 cm
Length of the tube	L	6 cm
Radius of the tube	r	0.5 cm
Fluid density	ρ	1 g/cm ³
Fluid viscosity	μ	0.01 g/cm·sec
Frequency	f (1/T)	0.05 Hz \sim 10 Hz
Compression duration	d	0.1 \sim 1
Amplitude	A_0	0.5 cm
Stiffness constant	κ_t	900 g/sec ² ·cm
Stiffness constant	κ_c	120 g·cm/sec ²

RESULTS AND DISCUSSION

The main results are as follows: First, a net flow inside the elastic tube is observed by the periodic compress-and-release action that is applied to an asymmetric location of the elastic tube. Second, the direction and magnitude of a net flow are dependent not only on the position of the driving function but also on the parameters. We focused on the crucial factors, frequency and compression duration, although it has been reported in many earlier works that the direction and magnitude are also determined by other parameters, such as the amplitude of the driving function, the radius and wall thickness of the tube, and the ratio of the length of the pumping position and the rest of the tube [17, 20, 21]. Third, there exist mostly one-directional net flows if a long thin tube is considered and the pumping is applied to the short portion near the edge of the tube. Fourth, the fundamental question of valveless pumping is explained by introducing the signed-area of a flow-pressure loop: why does a unidirectional net flow exist in a valveless pump system? Last, an elastic material without a stiffer material is enough to generate a net flow in a valveless pump system. Note that we have confirmed the first order accuracy in our numerical method.

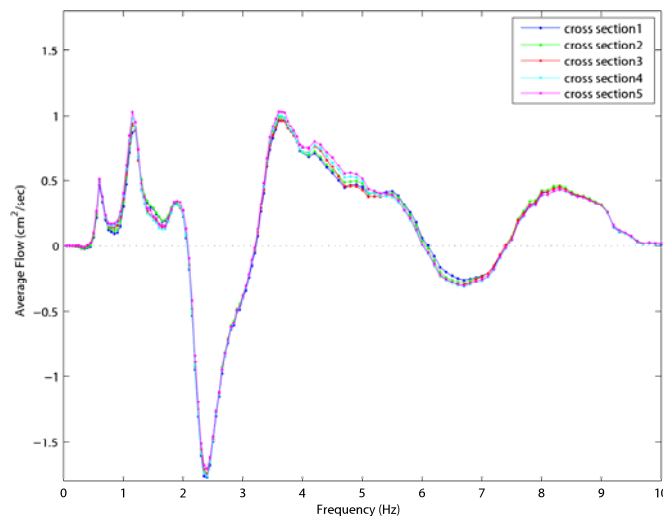


FIG 2. The space- and time-averaged flows as a function of the frequency.

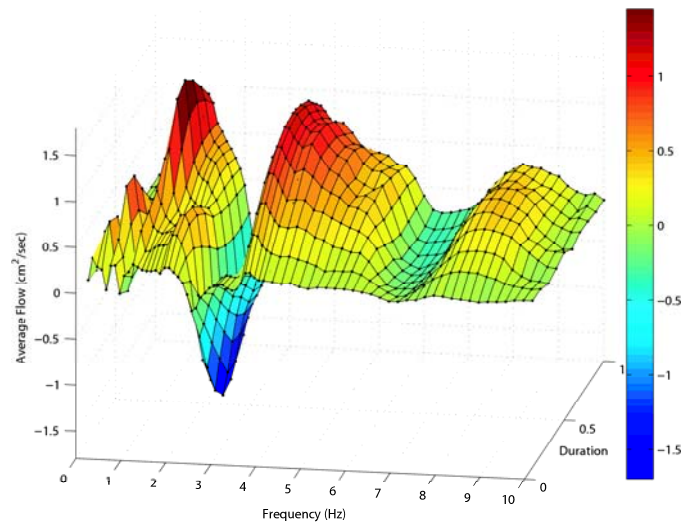


FIG 3. The space- and time-averaged flows as functions of the frequency and the compression duration.

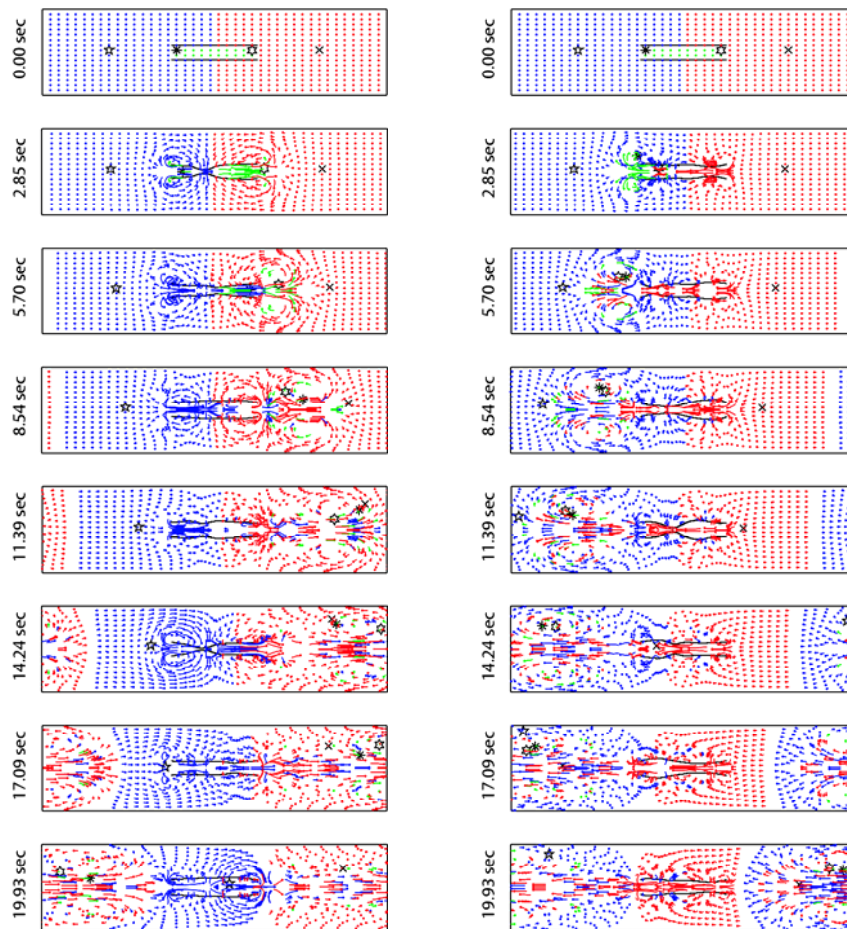


FIG 4. A positive and negative directional flows at $f=1.15\text{Hz}$ and $f=2.4\text{Hz}$

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